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Two-dimensional discrete wavelet analysis of multiparticle event topology in heavy-ion collisions

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Abstract
The event-by-event analysis of multiparticle production in high energy hadron and nuclei collisions can be performed using the discrete wavelet transformation. The ring-like and jet-like structures in two-dimensional angular histograms are well extracted by wavelet analysis both for toy models and in Monte Carlo simulated events of nucleus–nucleus collisions at LHC energies. For the first time the method is applied to the jet-like events. The jet positions are located quite well by the discrete wavelet transformation of angular particle distribution even in the presence of underlying soft process background in nucleus–nucleus collisions at LHC energies.

1. Introduction

Physics of high energy nucleus–nucleus collisions is much more complex than a mere independent superposition of nucleon–nucleon collisions. The first results from RHIC on AuAu collisions at $\sqrt{s} = 200A$ GeV already evidenced now some collective effects. The detailed analysis of RHIC results is presented in the review papers of all four experiments BRAHMS [1], PHOBOS [2], STAR [3], PHENIX [4]. It is concluded that the matter, which is formed, is not described in terms of a color-neutral system of hadrons. This state of matter undergoes the dense stage reminding a liquid. It is called quark–gluon liquid or strong See endnote 1 interacting quark–gluon plasma (sQGP) (e.g., see [5]). The manifestation of this new matter See endnote 2 can lead to unexpected effects such as multiple gluon minijet production, asymmetric and odd number of quark jets, and collective effects.

The most spectacular structure in the angular distribution of created particles is jets of hadrons well collimated along the jet axis. Comparison of jet production in AA and pp collisions or in the central and peripheral nucleus–nucleus collisions allows us to estimate jet absorption in the partonic matter. This is one of the main signatures of QGP (e.g., see [6]).

Among the suggested QGP signatures, the event-by-event topology plays a special role because fluctuations and correlations in angular particle distribution provide important information about the dynamics of a process. An example of such kind of structure is two-particle angular correlations in the phase space at low transverse momenta $p_T < 2$ GeV/c which shows that jet-like structure exists even in soft hadron correlations and depends on collision centrality [3]. The collective azimuthal flow is seen in individual events as their elliptic shape. The ring-like structure of the charged particle distribution in the pseudorapidity–azimuthal angle (η–φ) plane was first observed in the cosmic ray event analyzed in [7]. Large statistics experiments at RHIC with nucleus–nucleus central collisions revealed the similar structure around the away-side jets in two- and three-particle correlations [8, 9]. This phenomenon was interpreted as initiated by Cherenkov gluons [10–14] or Mach waves [15–19] induced by a parton in a medium. This is a new physical effect. It can be explained theoretically as the classical solution of the equations of the in-medium gluodynamics proposed in [20] or by hydrodynamics [5, 17]. In both cases the conical shock waves in the nuclear medium lead to the ring-like structure in the plane perpendicular to the cone axis.

The analysis of event structure by the wavelet transformation is very fruitful for such processes. The wavelet transformation was applied to both one- [21, 22] and two-dimensional [11, 23, 24] particle distributions in nucleus–nucleus collisions. The discrete wavelet transformation (DWT) [11] and continuous wavelet transformation [23, 24] were used. It was shown that the wavelet analysis allows us to reveal fluctuation patterns. The 'texture' of some events in AA collisions was investigated with the help of DWT [25, 26].

In the future experiments at LHC (large hadron collider) energies the jet topology of events can give the information about characteristics of sQGP, produced in nucleus–nucleus collisions. It is necessary to reconstruct energy and position of jets with good resolution. Jet finding algorithms of cone type select groups of particles within a cone of the certain (comparatively small) radius $R_{\rm jet} = \sqrt{(\Delta\eta)^2 + (\Delta\varphi)^2}$ in pseudorapidity–azimuthal angle (η–φ) space [27]. The cone-type algorithm cannot extract complex structures in two-dimensional angular distributions like rings. The jet particles fill in their cones with maximum at the cone axis while ring-like structures appear when particles are close to the cone surfaces. It is also impossible to distinguish a ring and a jet in the same event or to recognize the jets with different widths. We show that this can be done by the wavelet transformation in the event-by-event analysis of multiparticle production.

With this goal in mind we try to find if the wavelet transformation helps to answer the following questions:

(1) Is it possible to distinguish different structures in the same event analyzing the shape of their angular distributions?
(2) How to find and separate the hadron jets with different widths?
(3) What is a criterion of jet recognition over a strong background in heavy-ion multiparticle events?

We answer these questions by showing that the method of the wavelet analysis allows us to distinguish jets from rings and to find jet positions among the underlying backgrounds in Monte Carlo simulated events. It is also possible to decrease the low energy threshold of jet event reconstruction [28].

However, the detailed numerical study of the jet finding efficiency and jet position resolution by wavelet analysis is still beyond the scope of the present paper. The development of the wavelet jet finding algorithm is the problem for further studies.

The paper structure is as follows. The basic concepts of wavelet analysis are discussed in section 2. Jet and ring structures in the same toy-model event are analyzed by the two-dimensional discrete wavelet transformation in section 3. Three jets with different energies in one event plus the underlying soft background, simulated by PYTHIA and HYDJET event generators for CMS installation at LHC, are investigated in section 4. Section 5 contains our conclusions.

2. The discrete wavelet transformation

In many papers on wavelets [29–31] it is shown that for the expansion of a two-dimensional function $f(x,y)$ in a wavelet basis one can use the one-dimensional expansion coefficients. Therefore here we explain briefly the basic concepts for the expansion and restoration of the one-dimensional function $f(x)$.

There is a complete orthonormal basis of scaling $\{\varphi_{j,n}(x)\}$ and wavelet $\{\psi_{j,n}(x)\}$ functions with compact support in which one can expand any measurable function as a series

$$f(x) = \sum_{n=-\infty}^{\infty} a_{j_0}[n]\varphi_{j_0,n}(x) + \sum_{j=j_0} \sum_{n=-\infty}^{\infty} d_j[n]\psi_{j,n}(x). \tag{1}$$

One may choose wavelets which are defined by a finite number of real coefficients $h[n]$, called a filter. Coefficients $h[n]$ correspond to the scaling function $\varphi(x)$, and coefficients $g[n] = (-1)^{1-n}h[1-n]$ to the wavelet function $\psi(x)$. Then the orthonormal basis is constructed as a set of functions

$$\{\psi_{j,n}(x)\} = 2^{j/2}\psi(2^j x - n)\}_{j=0,1,\ldots} \tag{2}$$

The wavelet function $\psi_{j,n}(x)$ expands 2^j times with the scale changing and shifts along the x-axis by n. This allows us to find features of $f(x)$ not only on the whole x-axis (as in Fourier transformation) but also in narrow regions corresponding to smaller scales. The restoration is complete due to completeness and orthonormality of the wavelet basis. Further we follow Mallat [29].

A great advantage of the discrete wavelet analysis is the opportunity of proceeding with the so-called fast wavelet transformation (FWT) using the coefficients $h[n]$ and $g[n]$. Computing can be done by the iterative procedure and is therefore fast.

Values of the function $f(x)$ on the smallest scale are linked to the coefficients $a_{j,n} \equiv a_j[n]$, and further algorithm of transition to a larger scale is [29]:

$$a_{j+1}[p] = \sum_{n=-\infty}^{\infty} h[n-2p]a_j[n], \qquad d_{j+1}[p] = \sum_{n=-\infty}^{\infty} g[n-2p]a_j[n], \tag{3}$$

while the algorithm of restoration is

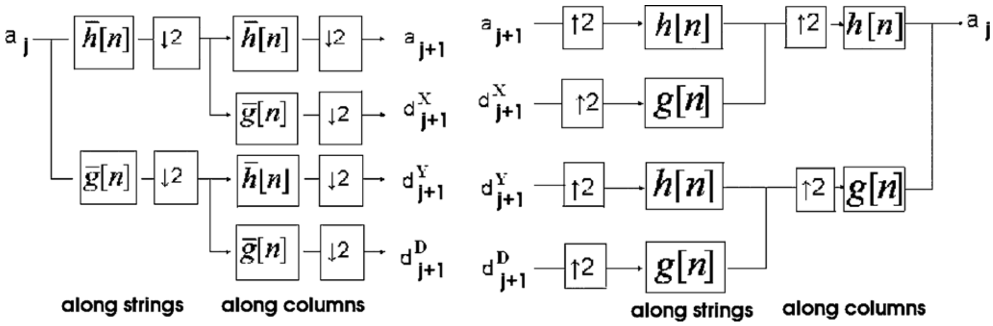
$$a_j[p] = \sum_{n=-\infty}^{\infty} h[p-2n]a_{j+1}[n] + \sum_{n=-\infty}^{\infty} g[p-2n]d_{j+1}[n]. \tag{4}$$

The sums in (3) and (4) are finite so the finite number of coefficients $h[n]$ and $g[n]$ is used.

In the two-dimensional case a separable form of wavelet functions is used [29]. Then the orthonormal basis is constructed as a set of functions $\left\{\Gamma^{\mathrm{X}}_{j,n_1,n_2}, \Gamma^{\mathrm{Y}}_{j,n_1,n_2}, \Gamma^{\mathrm{D}}_{j,n_1,n_2}\right\}$:

$$\begin{aligned}
\Gamma^{\mathrm{X}}_{j,n_1,n_2}(x_1,x_2) &= 2^j\varphi(2^j x_1 - n_1)\psi(2^j x_2 - n_2);\\
\Gamma^{\mathrm{Y}}_{j,n_1,n_2}(x_1,x_2) &= 2^j\psi(2^j x_1 - n_1)\varphi(2^j x_2 - n_2);\\
\Gamma^{\mathrm{D}}_{j,n_1,n_2}(x_1,x_2) &= 2^j\psi(2^j x_1 - n_1)\psi(2^j x_2 - n_2).
\end{aligned} \tag{5}$$

Separable two-dimensional convolution can be defined as a product of one-dimensional convolutions on the rows and columns. Symbolically, the decomposition and restoration of a two-dimensional function $f(x,y)$ is shown in figure 1 (see [29]). Here $\bar{h}[n] = h[-n]$. Decomposition (figure 1(a)) is done first along strings of table $a_j(x_1,x_2)$, then along columns. Restoration (figure 1(b)) is done in the opposite direction.



(a) (b)

Figure 1. Symbolic schemes of the two-dimensional DWT: (a) decomposition, (b) restoration.

3. Wavelet analysis of complex structures in two-dimensional distributions

The wavelet analysis is often called a mathematical microscope since it allows us to examine a signal with different resolution and to reveal structures on different scales. We demonstrate it here with some examples using the Daubechies wavelet D^8 [30]. DWT with the Daubechies wavelet D^8 was tested by the example of the one-dimensional function in [32].

In figure 2(a), the two-dimensional histogram of angular distribution in the $\eta \times \varphi$ space is simulated from $N = 100000$ entries, including the jet and ring-like shape as a toy example. The histogram binning is 128×128. The values in each bin of the histogram determine the coefficients $a_{j=0}[n_1, n_2]$ for the smallest scale $j = 0$. The whole interval of the histogram along both axes η and φ corresponds to the scale $j = 7$.

With the help of the wavelet analysis we try to reveal different forms of irregularities in the histogram. For this purpose we use the wavelet decomposition of the histogram and then restore its structure at different scales. If during restoration we set to zero the coefficients $d_j[n_1, n_2] = 0$ for large scales $j = 3, 4, 5, 6, 7$, then only a narrow peak is restored (figure 2(b)). If during restoration we set to zero the coefficients $d_j[n_1, n_2] = 0$ for small scales $j = 1, 2, 3$ then the ring-like structure, which initially is present in the distribution, is revealed and the narrow peak disappears (figure 2(c)).

Let us consider another example of the function containing two peaks of different shapes:

$$f(\eta, \varphi) = \frac{1}{\sqrt{(\eta - \eta_0)^2 + (\varphi - \varphi_0)^2}} + A\exp\left\{-0.5\frac{(\eta - \eta_1)^2 + (\varphi - \varphi_1)^2}{\sigma^2}\right\}. \tag{6}$$

The chosen parameters are $\sigma = 0.4$ and $A = 40$ (the particular values for peak positions are irrelevant and chosen for peaks not to overlap). The peaks have different widths. Therefore they correspond to different scales in wavelet expansion. Just as in the previous example, we produce the histogram from this function (figure 3(a)). If we leave only coefficients $d_j[n_1, n_2]$ for $j = 1, 2$ at small scales, we get the narrow peak (figure 3(b)). The wide peak is restored quite well if in the wavelet series we leave coefficients $d_j[n_1, n_2]$ at large scales $j = 4, 5, 6$ (figure 3(c)).

4. Wavelet analysis of jet events

Jet reconstruction by usual methods becomes difficult in central PbPb collisions at LHC energy [33] because of large background with $(\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}y)_{y=0} = 3000$–$5000$ charged particles.

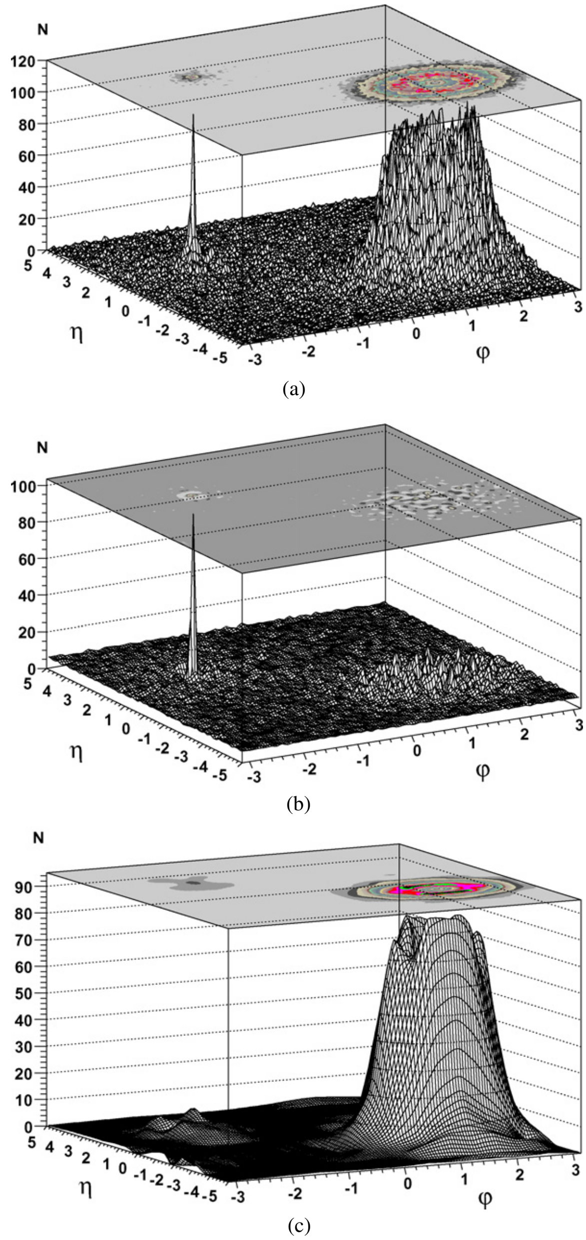
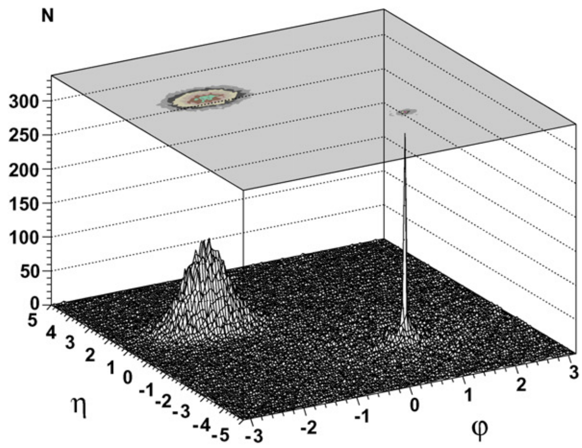
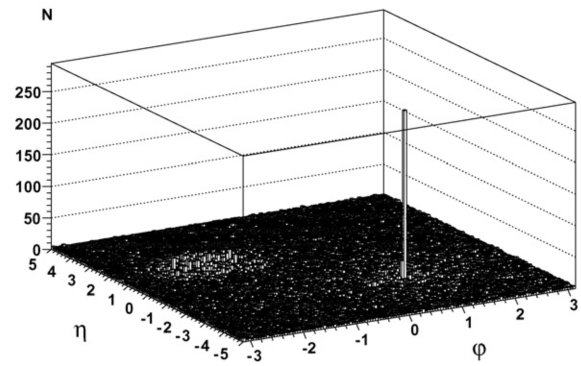


Figure 2. Ring-like and jet-like structures in angular particle distribution: (a) both structures, (b) wavelet restoration without large scale coefficients, $d_j[n_1, n_2] = 0$ at $j = 3, \ldots, 7$, (c) wavelet restoration without small scale coefficients, $d_j[n_1, n_2] = 0$ at $j = 1, \ldots, 3$. View in the upper plane of figures is the distribution projected on the plane.

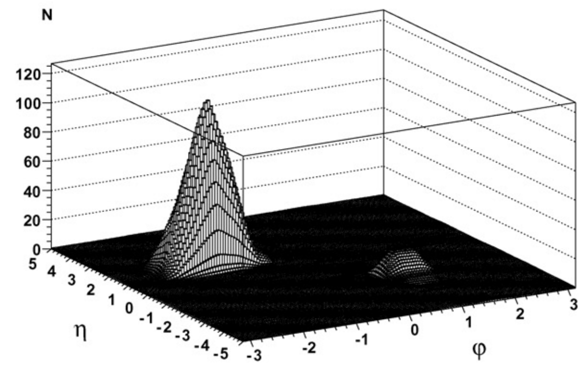
Therefore usual cone-type algorithms are modified in different ways to take into account this background. In [33, 34], the possibility of jet recognition in central PbPb collision in the CMS



(a)



(b)



(c)

Figure 3. Narrow and wide jets in angular particle distribution (6): (a) both jets, (b) wavelet restoration without large scale coefficients, $d_j[n_1, n_2] = 0$ at $j = 3, \ldots, 7$, (c) wavelet restoration without small scale coefficients, $d_j[n_1, n_2] = 0$ at $j = 1, \ldots, 3$.

detector with the help of a modified UA1-algorithm was studied. It is shown that for jets with $E_T > 50$ GeV the recognition efficiency is close to 100%.

Jets with different transverse momenta p_T were simulated for pp collisions at LHC energy $\sqrt{s} = 5500$ GeV by the PYTHIA event generator [35]. Then the angular distribution of particle transverse momentum was superimposed on the background, simulated by the HYDJET generator [33]. The background is calculated with multiplicity $(\mathrm{d}N/\mathrm{d}y)_{y=0} \approx 4000$ in the central rapidity region of PbPb collisions as obtained from a simple interpolation from RHIC to LHC. The jet production is switched off in the background events.

Histograms are made by binning of the two-dimensional distribution within intervals $|\eta| < 2.4$ and $|\varphi| < \pi$ with the number of bins 128 for each variable. The sizes of bins are equal to $\Delta\eta \times \Delta\varphi = 0.0375 \times 0.049$, which are close to CMS tracker detector granularities. Jets of hadrons are usually well collimated along the jet axis, so we search by the wavelet method for narrow peaks in the distribution of the transverse momentum $\frac{\mathrm{d}^2 p_T}{\mathrm{d}\varphi \mathrm{d}\eta}$. The wavelet analysis is used to find the position of jets. The background is treated as a large-scale structure (in our case $j = 2$–7).

The following algorithm of jet recognition with the help of the wavelet analysis is proposed:

(1) A given event distribution is decomposed by the wavelet discrete transformation.

(2) A smooth background at the large scale $j = 7$ is evaluated and then subtracted from the distribution.

(3) To remove the irregularities at the large scales the coefficients $d_j[n_1, n_2]$, $j = 4$–7 are set to zero. The range of small scales $j < 4$ corresponds approximately to the region $\sqrt{(\Delta\eta)^2 + (\Delta\varphi)^2} < R_{\mathrm{jet}} = 0.3$ from the center of the narrow peak.

(4) The coefficients at small scales $d_j[n_1, n_2]$, $j = 1, 2, 3$, which are below a certain threshold, are set to zero in order to remove the sharp irregularities with small intensity. These cuts correspond to conditions $d_j[n_1, n_2] < \mathrm{cut} \times (d_j[n_1, n_2])_{\mathrm{max}}$. The parameter cut $= 0.95$ was found as optimum among many others.

(5) The event distribution is restored with the selected $d_j[n_1, n_2]$ coefficients.

The event simulated according to the described procedure with three jets with total $p_T = 53$ GeV, $p_T = 45$ GeV and $p_T = 22$ GeV in their regions is shown in figure 4(a) before the background was added. The sum with the background is shown in figure 4(b) . The same event with jet reconstruction by the proposed method is shown in figure 4(c). Here the smooth background has been removed by setting to zero $d_j[n_1, n_2]$ at the largest scale $j = 7$. The sharp peaks with small intensity in the background are removed by the cuts of small scale $d_j[n_1, n_2]$-coefficients below the threshold. Position of jets is well allocated on position of peaks in the spectrum of wavelet coefficients at different scales depending on the width of a jet. Figure 4(c) shows that the peak intensities are decreased and their shapes are distorted.

Only two jets are restored in this case. The third jet with $p_T = 22$ GeV$/c$ is lost because it is compatible with background fluctuations. Another disadvantage of this selection is very strong distortion of the total transverse momentum values. The calculation of jet energy must be made by other methods (for example, by the cone method). But the jet positions are reconstructed quite well by DWT.

The background under the jet may be different and depend on the jet position in the η–φ plane. So it is not correct to subtract the background, calculated as an average background in the η–φ space. Here we propose to estimate the background contribution under the jet by calculating it inside a ring around the jet. Let us know the position of the jet by wavelet analysis and select some region of the jet as $\sqrt{(\Delta\eta)^2 + (\Delta\varphi)^2} < R_{\mathrm{jet}}$. Then we can calculate the total transverse momentum in the ring $R_{\mathrm{jet}} < \sqrt{(\Delta\eta)^2 + (\Delta\varphi)^2} < R_{\mathrm{out}}$. In our case we take R_{out} so that the number of bins under the jet and in the ring are equal.

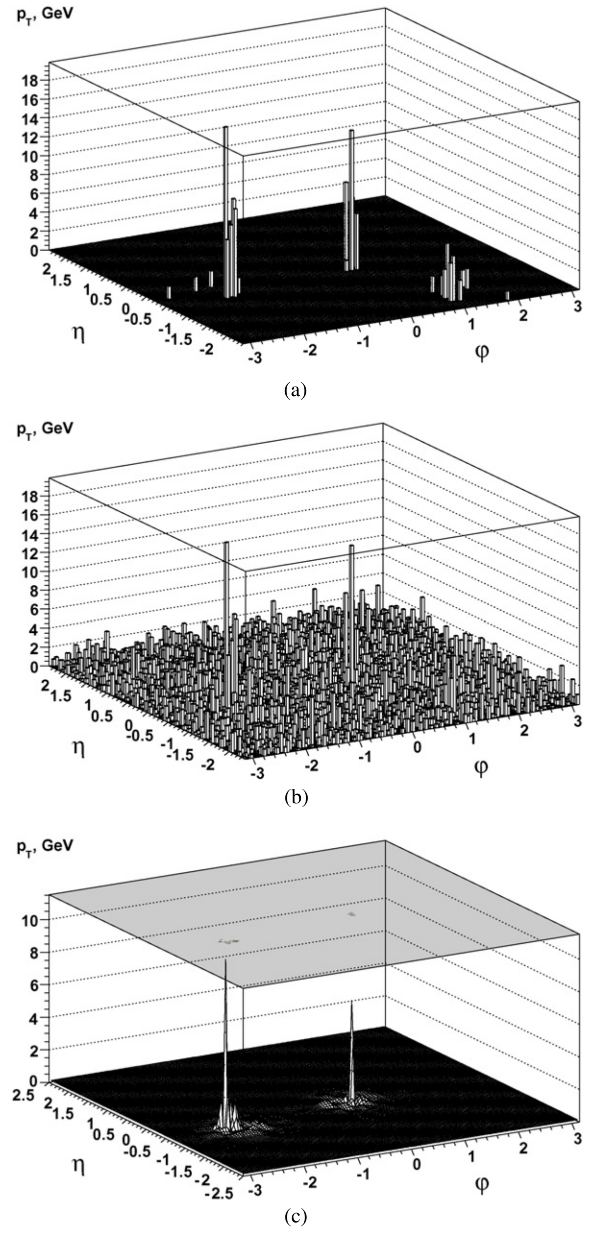


Figure 4. (a) The event with jet transverse momenta $p_T = 53$ GeV, $p_T = 45$ GeV and $p_T = 22$ GeV, simulated by PYTHIA, (b) sum of the event and the background, calculated by HYDJET, (c) the distribution of the event after restoration by the suggested method of jet searching.

Let us define the average transverse momentum $\langle p_T \rangle = \frac{1}{N\mathrm{bin}} \sum_{k=1}^{N\mathrm{bin}} p_{T,k}$, where N_{bin} is the number of bins in the region of jet or ring in the angular distribution. It is estimated separately for jets in PYTHIA and for the background in HYDJET. The energy dependence of the jet widths is not taken into account here. The first (53 GeV$/c$) and the second (45 GeV$/c$) jets have the average $\langle p_T \rangle$ equal to $\langle p_T \rangle_{\mathrm{jet}} = 0.346$ GeV$/c$ and 0.295 GeV$/c$.

They are larger than those for background $\langle p_T \rangle_{back} = 0.330$ GeV/c and 0.246 GeV/c. For the third jet (22 GeV/c) in figure 4(a) the average $\langle p_T \rangle_{jet} = 0.143$ GeV/c is less than for background ($\langle p_T \rangle_{back} = 0.209$ GeV/c). Thus the wavelet analysis cannot find this jet.

The best way to study jet events with large background by the wavelet method is as follows. First, one finds positions of jets by the suggested procedure. Then the background contribution is calculated in the ring region around each jet. The genuine energy of the jet is obtained by the subtraction of the background contribution from the total energy in the jet region.

5. Conclusions

Wavelet analysis allows us to find jets and ring-like structures in individual high multiplicity events in nucleus–nucleus collisions. We have tested it for some analytical toy-examples and for Monte Carlo simulated events at LHC energies. The two-dimensional discrete wavelet transformation reveals well the irregularities with different shapes by the corresponding choice of the wavelet scale j. Such structures correspond to different effects in nucleus–nucleus collisions. The ring-like structure is impossible to reveal by well-known jet algorithms, but this goal is achieved with the help of the wavelet analysis. The method is applied to the jet-like event with a high multiplicity background for the first time. The DWT allows us also to distinguish the bumps with different widths in the two-dimensional angular distribution in the same event. It is very important for diagnostics of quark and gluon jets.

The discrete wavelet transformation was tested on the MC simulated particle angular distributions, which are close to foreseen experimental LHC data. Position of jets is well allocated on position of peaks in the spectrum of wavelet coefficients at different scales depending on the width of a jet. Removing the wavelet coefficients at large scales allows us to subtract the smooth background. Removing the wavelet coefficients at small scales below a certain threshold allows us 'to clean out' event from the sharp irregularities with small intensity. One must be careful with excluding the background by wavelet analysis. Its subtraction can decrease the jet intensity (energy, number of particles) and change the jet shape. It can be recommended for use only to find the jet position. The jet energy must be estimated by other methods. However, the jet position is defined quite well by the discrete wavelet transformation. The detailed numerical study of the jet finding efficiency and jet position resolution by wavelet analysis is the task of nearest future.

At the end we would like to stress that the wavelet analysis can be used more widely for studies of the general topology of individual high multiplicity events. Beside jets and rings other structures can be found as was noted already. E.g., the common features of correlations in particle locations within the three-dimensional phase space such as fractality and intermittency can be revealed [36].

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Endnotes

(1) Author: Please check the use of the word 'reminding' in the sentence 'This state . . . reminding a liquid'.

(2) Author: Please check whether the edit in the sentence 'The manifestation of this new matter . . . and collective effects.' retains the intended sense.

(3) Author: Please be aware that the colour figures in this article will only appear in colour in the Web version. If you require colour in the printed journal and have not previously arranged it, please contact the Production Editor now.

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