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Decay Amplitudes with t' Dependence:

Consider a process

$$a + b \to c + d, \qquad c \to 1 + 2 \tag{1}$$

The usual variable t' is defined through

$$t' = 2\, p_a\, p_c\, (1 - \cos\theta_0) \tag{2}$$

where p_a, p_c and $\cos\theta_0$ are given in the overall center-of-mass (CM) frame. The decay amplitude evaluated in the c rest frame (the c RF) is given by

$$A^{\xi}_{\lambda_1\,\lambda_2}(m,\Omega) = \sqrt{\frac{2j+1}{4\pi}}\, F^{\xi}_{\lambda_1\,\lambda_2}\, D^{j\,*}_{m\,\delta}(\phi,\theta,0), \quad \delta = \lambda_1 - \lambda_2 \tag{3}$$

where $\xi = \{j, \eta\}$ with η standing for the intrinsic parity of c and $|jm\rangle$ is the spin state for c before its decay, and $\Omega = (\theta,\phi)$ are the appropriate decay angles of the particle 1 in the c RF. $F^{\xi}_{\lambda_1\,\lambda_2}$ is the usual helicity-coupling decay amplitude. The relevant axes for the decay frame (the Jackson frame) are defined as follows: the z axis is given by $\vec{p}_c$, and the y axis by $\vec{p}_a \times \vec{p}_c$, all in the overall CM frame, i.e.

$$\vec{z} \propto \vec{p}_c, \quad \text{and} \quad \vec{y} \propto \vec{p}_a \times \vec{p}_c \tag{4}$$

If $\theta_0 = 0$ or $t' = 0$, then the reaction becomes collinear, i.e. the reaction plane for the overall CM frame does not exist. As a consequence, the y axis is *not* defined, since $\vec{p}_a$ is parallel to $\vec{p}_c$ [see (4)]. So the decay amplitude A becomes indeterminate, as the angle ϕ for $m \neq 0$ cannot be defined [see (3)]. It is clear, therefore, that the phenomenological decay amplitude corresponding to the process (1) must be modified to include a dependence on t'. For the purpose, we introduce a new function of t'

$$\begin{cases} Q_m(t') = \exp(1-v), \qquad m = 0 \quad \textit{and} \quad v = \dfrac{t'}{t'_0} \\ Q_m(t') = v\,\exp(1-v), \qquad m \neq 0 \end{cases} \tag{5}$$

where t'_0 is a parameter to be determined experimentally. The functions are normalized such that $Q_m(t'_0) = 1$, and they have the desired properties

$$\begin{cases} Q_m(0) = \mathrm{e} \quad \text{for} \quad m = 0 \\ Q_m(0) = 0 \quad \text{for} \quad m \neq 0 \end{cases} \tag{6}$$

The modified decay amplitudes are

$$A^{\xi}_{\lambda_1\,\lambda_2}(t',m,\Omega) = \sqrt{\frac{2j+1}{4\pi}}\,Q^{1/2}_m(t')\,F^{\,\xi}_{\lambda_1\,\lambda_2}\,D^{j\,*}_{m\,\delta}(\phi,\theta,0) \tag{7}$$

The function $Q_m(t'_0)$ is a unit-less quantity, which is damped at large values of v, and it ensures that $A = 0$, if $t' = 0$ *and* $m \neq 0$. The constant t'_0 is the inverse of the slope parameter. In principle, its value could depend on each ξ. However, it may be sufficient—in practice—to allow for two different values, corresponding to natural- and unnatural-parity exchanges. So we have

$$\begin{aligned}
&\frac{d\sigma}{dt'} \propto\ v\,\exp(1-v),\ \ (m \neq 0);\\
&\qquad v = \frac{t'}{t'_0}\quad \text{where}\quad t'_0 = \text{Parameter for Natural-Parity Exchange}\\
&\frac{d\sigma}{dt'} \propto\ \exp(1-v'),\ \ (m = 0)\quad \text{or}\quad \frac{d\sigma}{dt'} \propto\ v'\,\exp(1-v'),\ \ (m \neq 0);\\
&\qquad v' = \frac{t'}{t''_0}\quad \text{where}\quad t''_0 = \text{Parameter for Unnatural-Parity Exchange}
\end{aligned} \tag{8}$$

Note that there is just one t'-dependent form for the decay amplitudes with natural-parity exchanges, because the amplitudes are zero for $m = 0$. It should be borne in mind that the form of the function $Q_m(t'_0)$ is not unique, but it is merely the simplest possible in a phenomenological approach. As $t' \to 0$, the values of the decay amplitude would become unstable for $m \neq 0$, without the factor $\sqrt{v}$ which has the effect of reducing the magnitude of the unstable values; this is certainly a desirable attribute for the amplitudes in maximum-likelihood analyses.

Let the particles 1 and 2 be pseudoscalars, i.e.

$$A^{\xi}_{00}(t',m,\Omega) = Q^{1/2}_m(t')\,F^{\,\xi}_{00}\,Y^{\,j}_m(\theta,\phi) \tag{9}$$

and let there be three decay amplitudes, corresponding to j =0, 1 and 2. Then we obtain for $t' = 0$, from (7),

$$\begin{aligned}
A^{\xi}_{00}(0,m,\Omega) &= 0, \qquad \text{for} \qquad P_\pm \quad \text{or} \quad D_\pm\\
&= \mathrm{e}\,F^{\,\xi}_{00}\,Y^{\,j}_{00}(\theta), \qquad \text{for} \qquad S_0, \quad P_0 \quad \text{or} \quad D_0
\end{aligned} \tag{10}$$

As expected, the ϕ dependence is absent, if $t' = 0$.

In conclusion, an argument has been given for introducing a t' dependence in the decay amplitudes. The modified forms of the decay amplitudes should be better suited in analyses involving the maximum-likelihood methods than those *without* t'. Finally, one may introduce a more succinct form for the function

$$Q_m(t') = v^{|m|}\,\exp(1-v) \tag{11}$$

There is some justification in introducing the powers of $|m|$ to the basic functional form $\sqrt{v}$, as the ϕ dependence in the decay amplitudes involves higher polynomials of $\cos\phi$ and $\sin\phi$ with the increasing values of m. However, it could be argued that the two well-defined form of the t' dependence, given in (8) corresponding to natural- and unnatural-exchanges, might be preferable in a phenomenological approach.