Mass dependent fit of PWA results for $\eta\pi^0$ system
Part 2

V.L.Korotkikh, L.V. Malinina

26 June 2005

Scobeltsyn Institute of Nuclear Physics
Moscow State University, 119992 Moscow, Russia

Abstract

Mass dependent fit of PWA results of $\eta\pi^0$ system, which were received in each mass bin, are presented. Random selection of ambiguous solutions is used for the estimation of resonant parameter systematical errors. The fit in two t' - intervals of PWA shows a stability of fit results.

1 Description of the fit procedure.

In our $\eta\pi^-$ publication [1] we used a fit of two wave intensities and their relative phase. If we have in each mass (M is a center of mass bin) bin the results of PWA, for example the intensity of two wave $|V_1(M)|^2$, $|V_2(M)|^2$ and their relative phase $\Delta(1,2,M)$, then we can fit these distributions by the functions

$$V_1(M) = a_1 e^{i\alpha} f_1^{BW}(M) \tag{1}$$

$$V_2(M) = a_2 f_2^{BW}(M) \tag{2}$$

$$\Delta(1,2,M) = \alpha + \delta_1(M) - \delta_2(M) \tag{3}$$

Here are the Breit-Wigner (BW) amplitudes of resonance

$$f_j^{BW}(M) = \frac{M_j\Gamma_j}{M_j^2 - M^2 - iM_j\Gamma_J} = e^{i\delta_j(M)} sin\delta_j(M), \tag{4}$$

the phase of each wave is calculate by formula

$$ctg\delta_j(M) = \frac{M_j^2 - M^2}{M_j\Gamma_j} \tag{5}$$

and α is a constant production phase. Of course, Γ_j is a function of mass M with the corresponding barrier factor (see [1]). Here we omit the notation $\Gamma_j(M)$ for simplicity.

In the usual procedure of BW fit of two waves the intensities of two waves and phase difference between them are fitted simultaneously.

2 Results with the average ambiguous solutions.

The results of the PWA fit of 14188 events in the range $1.1 < M(\eta\pi^0) < 1.74$ GeV/c^2 and $0 <| t |< 1$ (GeV/c)^2 predicted by PWA fit for D_+ and P_+ waves (black points) and the result of the fit (solid lines) are in Fig.1 and Fig.2. The resonance parameters of $a_2(1320)$ and $\pi_1(1400)$ with their statistical errors are the next:

for the 2^{++} state (a_2^0 meson)

$$M_1 = M(a_2^0) = (1320 \pm 3)\ \mathrm{MeV/c^2}, \Gamma_1 = \Gamma(a_2^0) = (96 \pm 3)\ \mathrm{MeV/c^2}, \tag{6}$$

and for the exotic 1^{-+} state (π_1^0 meson)

$$M_2 = M(\pi_1^0) = (1270 \pm 14)\ \mathrm{MeV/c^2}, \Gamma_2 = \Gamma(\pi_1^0) = (334 \pm 42)\ \mathrm{MeV/c^2}. \tag{7}$$

$$\chi^2/DoF = 1.22$$

These results are obtained with averaged ambiguous solutions.

Ambiguous solutions.

There are eight ambiguous solutions in our PWA fit with seven waves for $\eta\pi^0$ system ([2],[3]). These solutions corresponds exactly to the same angular distributions.

In some cases it is possible to chose the physical one from them [4] using the predicted by Regge-pole model behavior of the ratio of the $D-$ and $D0$ waves intensities to the $D+$ intensity:

$$R = (I_{D-} + I_{D0})/I_{D+}.$$

In our case the statistical errors are large and the ratios for different solutions overlaps Fig. 3. So in our case with not large statistic we failed to use the ratio R for the selection of physical solution.

Fig. 4 shows the ambiguous solutions for all waves. We calculate the integral intensities of D_+ and P_+ waves in the range of $1.24 < M(\eta\pi^0) < 1.34$ GeV/c^2. The ratio of P_+ and D_+ wave intensities for average solutions is equal to

$$R = |P_+|^2/|D_+|^2 = 0.35 \pm 0.1$$

3 The systematic studies.

For estimation of the systematical errors due to ambiguous solutions the following procedure was applied:

1. In each mass bin one solution was chosen randomly at large numbers $\simeq 10^3$ combinations .
2. The simultaneous fit of intensities of D+ and P+ waves and phase difference was done for the selected solutions in mass interval $1.1 < M(\eta\pi^0) < 1.74$ GeV/c^2.
3. The systematic errors of π_1^0 are calculated with fixed a_2^0 parameters.

We estimated the systematic errors for the a_2^0:

$$M(a_2^0) = (1320 \pm 3^{+10}_{-7})\ \mathrm{MeV/c^2}, \Gamma(a_2^0) = (96 \pm 3^{+40}_{-15})\ \mathrm{MeV/c^2}, \tag{8}$$

The mass of a_2^0 correspond to the PDG. A width includes E852 resolution. In the random procedure described above the mass and width of a_2^0 was taken from the resolution analysis of a_2^- in $\eta\pi^-$ - system [5] as $M = 1320$ MeV/c^2, $\Gamma = 118$MeV/c^2 and fixed. Fig.5 shows the ranges of possible values of mass and width of π_1^0 obtained with the random procedure. Fig.6 is the same as Fig.5, but with some selections described below.

The π_1^0 parameters with the systematical errors obtained with such procedure are the following:

$$M(\pi_1^0) = (1270 \pm 14^{+80}_{-70})\ \mathrm{MeV/c^2}, \Gamma(\pi_1^0) = (334 \pm 42^{+226}_{-184})\ \mathrm{MeV/c^2} \tag{9}$$

Large upper systematic error of $\Gamma(\pi_1^0)$ in (9) follows a long tail of $\Gamma(\pi_1^0)$ distribution in Fig.5b. The random correlation between the width and its error is shown in Fig.7. There are some points with errors larger then $\Delta\Gamma(\pi_1^0) > 50$ MeV. In the procedure of averaging of the ambiguous solutions the solutions with large errors give smaller contributions then the solutions with small errors. That is why the mean values of the resonance parameters are shifted toward small widths and masses.

If we take a cut $\Delta\Gamma(\pi_1^0) > 50 MeV$ (see Fig.7), then the distribution of $\Gamma(\pi_1^0)$ becomes narrow (see Fig.6b), but mass distribution is the same as in Fig.6a. The upper systematic error is decreased from 226 MeV to 116 MeV.

So the best estimation of π_1^0 parameters became

$$M(\pi_1^0) = (1270 \pm 14^{+80}_{-70})\ \mathrm{MeV/c^2}, \Gamma(\pi_1^0) = (334 \pm 42^{+116}_{-184})\ \mathrm{MeV/c^2}. \tag{10}$$

We show in [7] that the mass dependent partial wave analysis (MDPWA) of $\eta\pi^0$ system, which doesn't depend on ambiguous solutions, gives the very close π_1^0 parameters

$$M(\pi_1^0) = (1283 \pm 7)\ \mathrm{MeV/c^2}, \Gamma(\pi_1^0) = (382 \pm 24)\ \mathrm{MeV/c^2}. \tag{11}$$

4 Leakage study.

Sensitivity to leakage from D_+ wave.

In [1] was shown that a pure D+ wave can introduce a P+ wave artificially due to acceptance effects, resolutions. This "leakage" leads to a P+ wave that mimics the D+ intensity. In our case it has therefore the shape of $a_2(1320)$. The phase differences doesn't depend of mass. These features of "leakage from D+ wave" allowed us to introduce in the fit formula the member describing leakage as the "intensity of leakage" (I_{leak}) multiplied on the form of mass dependence of D+ wave with known $a_2(1320)$ resonance parameters. The intensity found of leakage from D_+ wave:

$$I_{leak} = 0.001 \pm 0.047$$

$$I_{leak}/I_{D+} \sim 10^{-7}.$$

These results are obtained with averaged ambiguous solutions.

We studied the systematical errors of π_1^0 parameters in the presence of leakage from D+ wave. The parameters of fit with including of D_+ wave leakage are in Table.1.

It was found (Fig.8) that presence of leakage makes more narrow π_1^0 masses distribution and shift width distribution to low values compared with Fig.5. But the upper and bottom limit of systematic errors are insignificantly changed.

Table 1. Results of the mass dependent fit with the average solutions.

Parameters	no leakage	leakage from $D+$	leakage from $S0$ and $D+$
M_1	1320 ± 3	1320 ± 3	1320 ± 2
Γ_1	96 ± 3	96 ± 3	101 ± 3
M_2	1270 ± 14	1270 ± 14	1273 ± 17
Γ_2	334 ± 42	334 ± 42	412 ± 57
α	-1.19 ± 0.09	-1.19 ± 0.09	-1.27 ± 0.08
I_{leakD+}	-	0.001 ± 0.07	0.001 ± 0.01
I_{leakS0}	-	-	569 ± 273
χ^2/DoF	1.22	1.25	1.19

Sensitivity to leakage from S0 wave.

We studied the systematical errors of π_1^0 parameters in the presence of leakage from S_0 waves. The leakage from S_0 wave was introduced by adding the member describing leakage as the "intensity of leakage" I_{leak} multiplied on the the form of mass dependence of S_0 wave with known $a_0(980)$ resonance parameters. The results with the leakage from S_0 and D_+ waves in the mass range was $0.8 < M(\eta\pi^0) < 1.74$ GeV/c^2 are present in Fig.9. One can see that the leakage from D+ wave is still negligible, but the leakage from S0 wave is large (50% of S0 intensitity):

$$I_{leakS0} = 569 \pm 273$$

$$I_{leakS0}/I_{S0} \sim 0.5$$

The new results of the resonance parameters are rather closed to the previous results (see Table 1). The systematical errors study gave the same ranges as in (9) and (10), because the leakage from S_0 wave is localized at mass range < 1100 MeV/c^2.

5 The non-resonant hypotheses in P_+ wave .

We try to describe a bump in P_+ wave intensity with the non-resonant $P+$ wave and the constant production phase. The parameters with and without leakage from D_+ wave are in a table.2. The obtained $\chi^2/DoF > 3$. It is larger then in the case of the resonant $P+$ wave , $\chi^2/DoF = 1.22$ (Table 1) .

In this case the relative phase form of mass dependence is only a_2-resonance phase. One can see that the "non-resonant" hypothesis fails to describe the phase difference between $D+$ and $P+$ waves (compare Fig.10 and Fig.9).

It is a strong argument for the resonant nature of P_+ wave bump.

Table 2. Results of the mass dependent fit with the average solutions if non-resonant hypothesis is considered.

Parameters	no leakage	leakage from $D+$
M_1	1313 ± 4	1315 ± 2
Γ_1	171 ± 10	222 ± 8
M_2	1301 ± 24	820 ± 14
Γ_2	358 ± 136	60 ± 58
α	-3.19 ± 0.06	-3.16 ± 0.04
$I_{leak_{D+}}$	-	1612 ± 227
χ^2/DoF	3.02	3.49

6 "Stability" of the resonance parameters.

Independence of the resonance parameters on the fit procedure.

One can see that the statistical errors of P_+ wave intensity are large (see Fig. 1). We produce the test aimed to answer the question: how much the large statistical uncertainties in P_+ wave intensity influence the fit results. The phase difference between $D+$ and $P+$ waves is measured with a good accuracy, whereas the intensity in $P+$ wave is measured with big statistic uncertainties.

We produced the new algorithm that makes the fit of intensity only one wave $|V_1(M)|^2$ and the relative phase of two waves:

1. The intensity of the first wave and phase difference between waves are fitted.
2. The optimal parameters of the both resonances and the intensity of the first resonance are found and fixed (M_1, Γ_1, M_2, Γ_2, $|a_1|^2$.
3. Then the intensity of the second wave ($|a_2|^2$) is fitted.

This fit procedure ("one wave and relative phase") suit very well the resonance structure in P_+ wave of $\eta\pi^0$ system. The parameters of the fit are in Table 3.

The results of the new fit procedure (Fig.2) are very close to the standard results (Fig.1). The differences in widths of $a_2(1320)$ and π_1^0 are in the statistical error limits.

Table 3. Results of the two fit procedures.

Parameters	Standard fit (Table 1)	"One wave and relative phase"
M_1	1320 ± 3	1322 ± 3
Γ_1	96 ± 3	102 ± 5
M_2	1270 ± 14	1270 ± 14
Γ_2	334 ± 42	332 ± 60
α	-1.19 ± 0.09	-1.2 ± 0.14
χ^2/DoF	1.22	0.73

We study the systematical errors for this type of fit by the method described above. The ranges of the possible values of mass and width of π_1^0 are smaller then for the usual type of fit because some correlations between parameters are absent, so the systematical errors are the same.

Independence of of t'-range.
In order to verify the independence of the fit procedure on the t'-range the large bin size 100 MeV was chosen. The results of the PWA fit of 23492 events in the whole t'-range and mass range $0.74 < M(\eta\pi^0) < 1.74 \quad (\mathrm{GeV/c})^2$ was compared with the results of PWA fit of 12062 events in $t' < 0.225 \quad \mathrm{GeV/c^2}$ range and 11430 events in $t' > 0.225 \quad \mathrm{GeV/c^2}$ range table 4. We were forced to use all mass range because the luck of statistics, so the leakages from D_+ and S_0 waves were included.

Table 4. Results of the mass dependent fit in different t'-ranges with 100 MeV mass bin.

Parameters	all region of t'	$t' < 0.225 \quad \mathrm{GeV/c^2}$	$t' > 0.225 \quad \mathrm{GeV/c^2}$
M_1	1327 ± 5	1320 ± 12	1326 ± 6
Γ_1	146 ± 8	151 ± 30	142 ± 8
M_2	1286 ± 40	1246 ± 51	1364 ± 50
Γ_2	585 ± 92	507 ± 117	612 ± 97
I_{leak_D+}	0.001 ± 0.1	0.001 ± 0.1	0.001 ± 0.1
I_{leak_S0}	333 ± 732	407 ± 592	0.001 ± 0.1
χ^2/DoF	1.3	0.83	1.1

The results predicted by PWA fit for D_+ and P_+ waves (points) and the result of the fit (lines) are in Fig.11 and Fig.12. You can see that the resonance parameters in different t'-ranges are consistent (Table 4). They coincide with the errors obtained with bin size 40 MeV in the statistical error limits. The leakage from S_0 wave is large at small t and is almost absent at high t'. It may be due to the dominant production of $a_0(980)$ meson at small t' by the b_1 -reggion exchange.

7 Conclusion

The resonance parameters of $a_2(1320)$ and $\pi_1(1400)$ with their statistical and systematical errors are the next:

$$M(a_2^0) = (1320 \pm 3^{+10}_{-7}) \ \mathrm{MeV/c^2}, \Gamma(a_2^0) = (96 \pm 3^{+40}_{-15}) \ \mathrm{MeV/c^2}, \qquad (12)$$

and for the exotic 1^{-+} state (π_1^0 meson)

$$M(\pi_1^0) = (1270 \pm 14^{+80}_{-70})\ \mathrm{MeV/c^2}, \Gamma(\pi_1^0) = (334 \pm 42^{+116}_{-184})\ \mathrm{MeV/c^2} \tag{13}$$

The parameters of mass and width are stable in the various fits and don't depend on t'.

References

[1] S.U.Chung et al., Phys./ Rev. D **60** 092001 (2001).

[2] S.A.Sadovsky, "On the Ambiguities in the partial wave analysis of $\pi^- p \to \eta\pi^0 n$ reaction", Inst. for HEP, IHEP-91-75 (1991).

[3] S.U.Chung et al., Phys./ Rev. D **56** 7299 (1997).

[4] S.A.Sadovsky, Nucl. Phys. A **655**) 131 (1999.

[5] K. Danyo, D. Weygand, "Resolutions Studies for the $a_2(1320)$", E852 note, 1997

[6] V.L. Korotkikh, L.V. Malinina, "Analysis of $\eta\pi^0$ system with the decay $\eta \to \pi^+\pi^-\pi^0$. Part 1", 2005.

[7] V.L. Korotkikh, L.V. Malinina, "Mass dependent partial wave analysis of $\eta\pi^0$ system. Part 3", 2005.

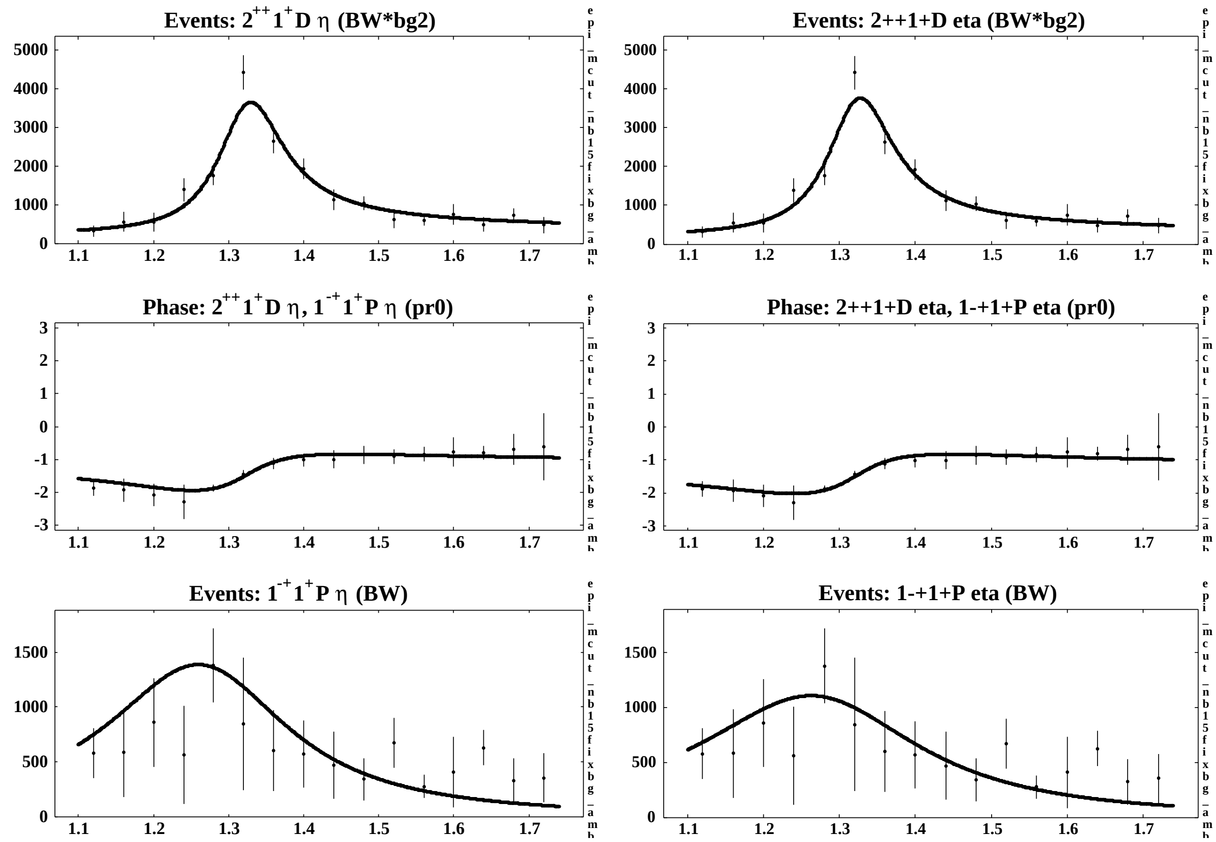


Figure 1: The standart fit of D+ and P+ wave intensities and phase difference between them.

Figure 2: The fit of intensities of D+ wave and phase between D+ and P+ waves. Then the mass and width of the resonant D+ and P+ waves were fixed and the intensity of P+ wave was fitted.

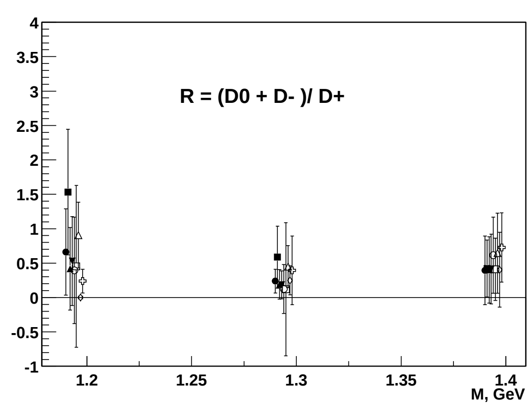


Figure 3: The ratio of the intensities of the unnatural exchange waves $|D_-|^2$ and $|D_0|^2$ to the natural $|D_+|^2$ in 100 MeV mass bin.

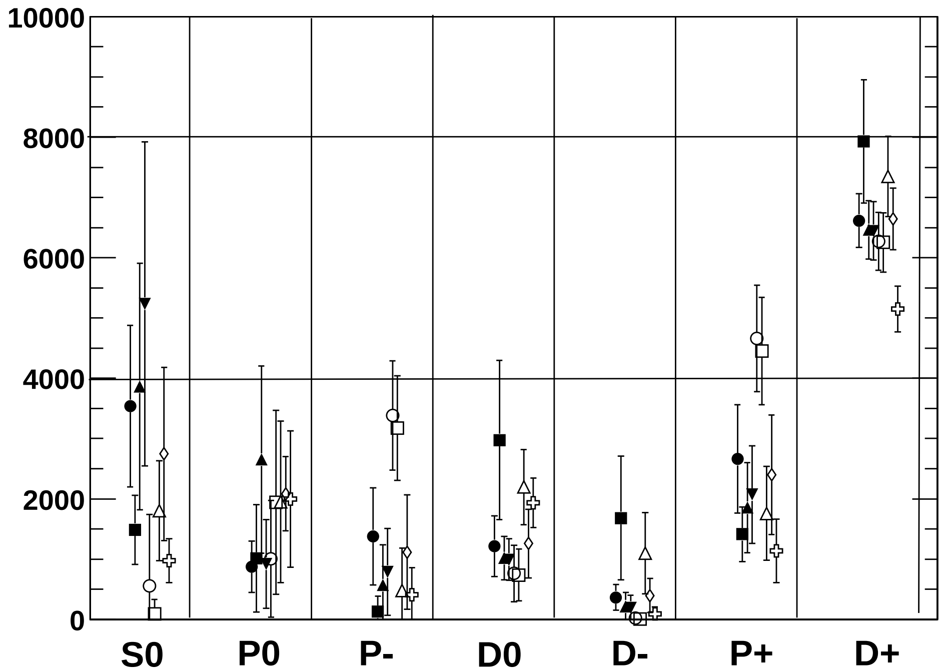
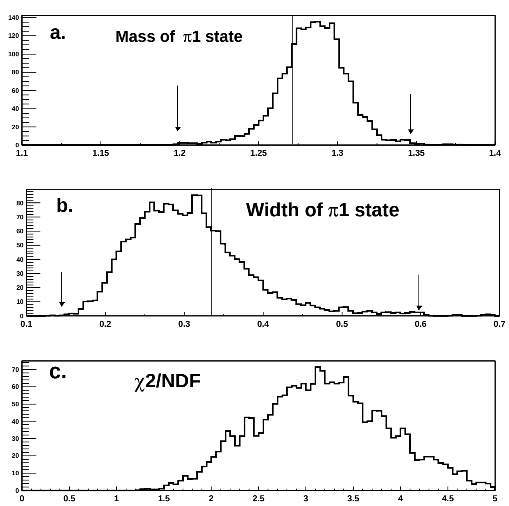


Figure 4: Ambiguous solutions of waves in mass bin (1.24 -1.34)GeV.



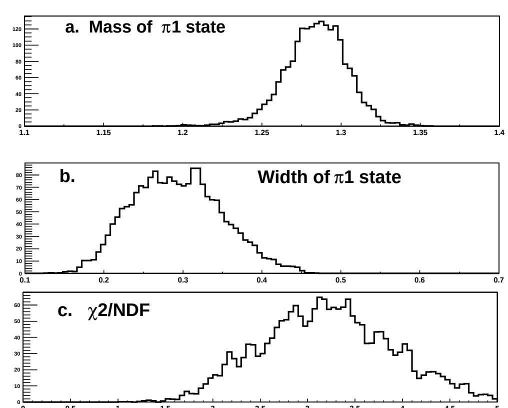


Figure 5: **a:** Mass and **b:** width ranges of π_1^0 state for solutions which have been randomly selected in each mass bin; **c:** χ^2/NDF. The arrows indicate the interval for calculation of systematic errors.

Figure 6: The same as in Fig. 5, but with cut of the width errors $\Delta\Gamma(\pi_1^0) < 50 MeV$.

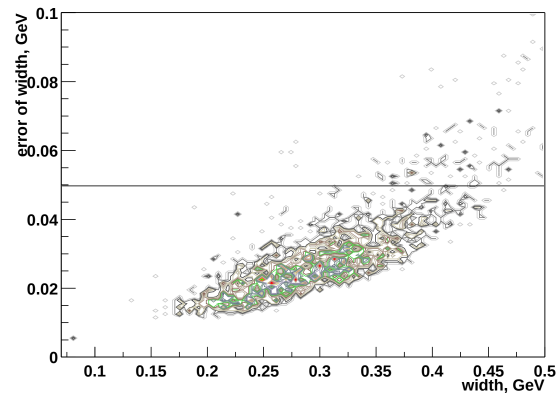


Figure 7: Correlations between a width $\Gamma(\pi_1^0)$ and error $\Delta\Gamma(\pi_1^0)$of the width for random solutions chosen .

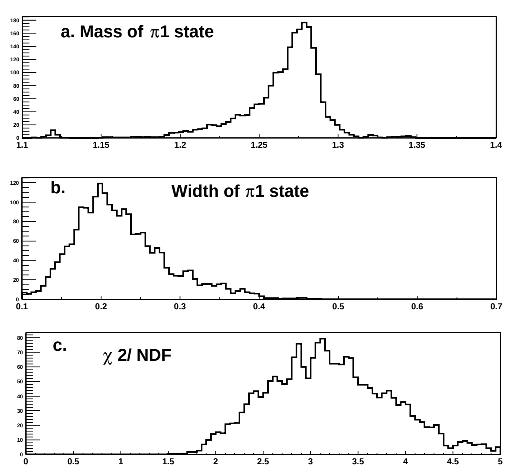


Figure 8: The same as in Fig. 5 with leakage from D_+ wave included.

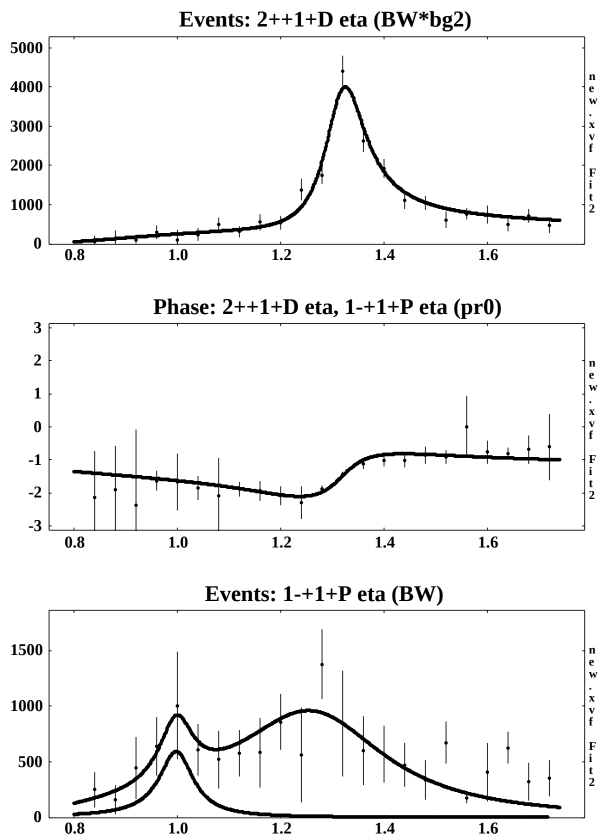


Figure 9: The fit of the intensities of D_+ and P_+ waves and phase between them with leakage from S_0 and D_+ waves. The parameters are in Table 4.

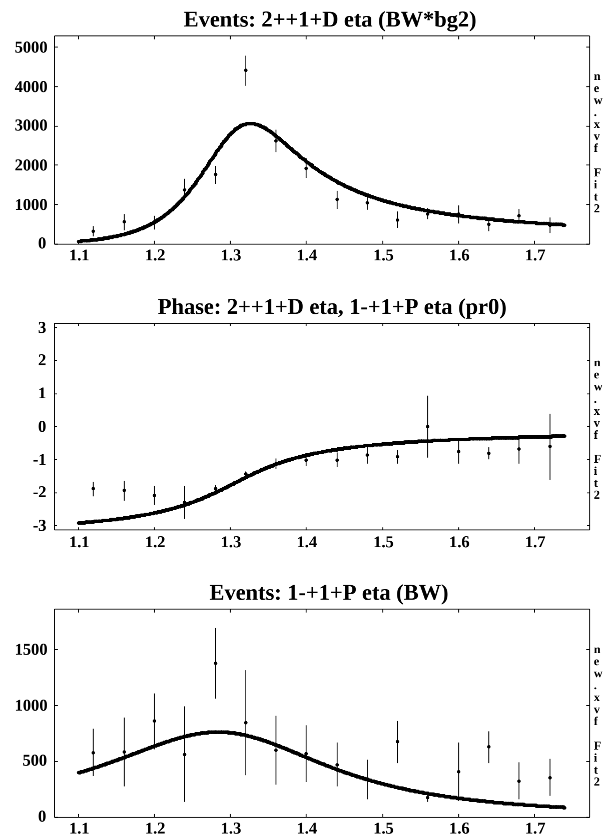
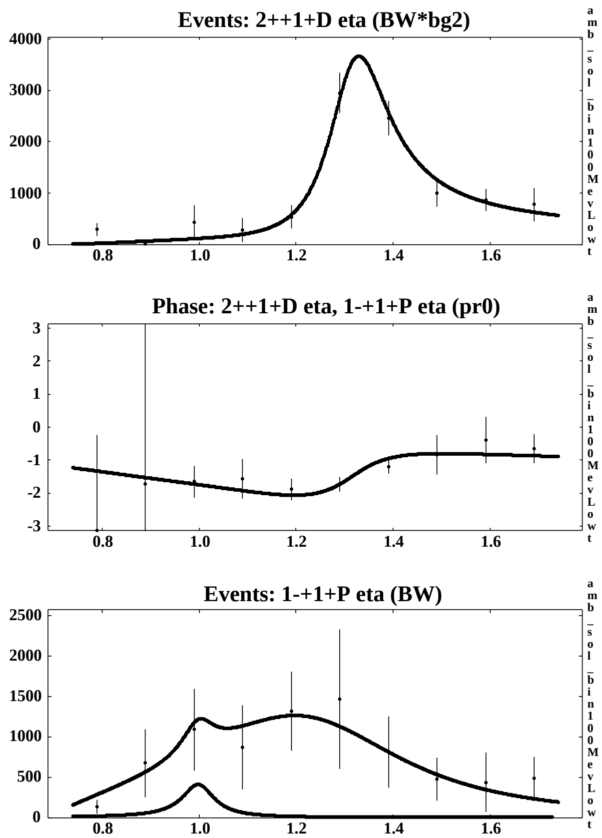


Figure 10: The results of fit of the intensities of D_+ and P_+ waves and phase between them with the non-resonance hypotheses of P_+ wave. The parameters are in Table 2.



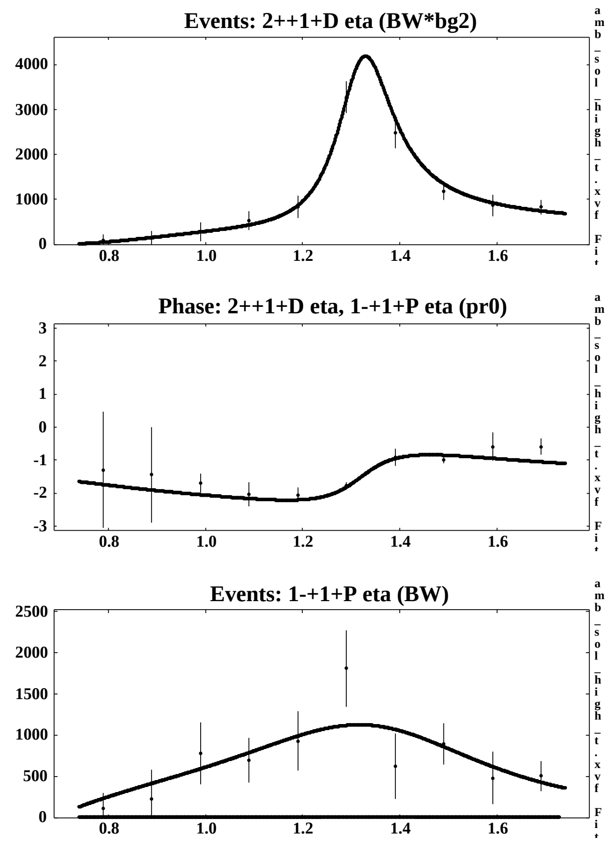


Figure 11: The fit of the intensities of D_+ and P_+ waves and phase between them at $0 < t' < 0.225(GeV/c)^2$.

Figure 12: The same as in Fig.11, but at $0.225 < t' < 1.0(GeV/c)^2$