Mass dependent partial wave analysis of $\eta\pi^0$ system
Part 3

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Abstract

The results of Mass Dependent PWA are demonstrated in the reaction $\pi^- p \to \eta\pi^0 n$, $\eta \to \pi^+\pi^-\pi^0$ at 18 GeV/c. Mass dependent partial wave analysis (MDPWA) is a fit, in which PWA of the angular $\eta\pi^0$ distribution is carried for each mass $\eta\pi^0$. The fitted functions are the Breit-Wigner amplitudes for the D_+, P_+ and some simple functions of mass for other waves. The results of MDPWA don't depend on the ambiguous solutions. The results of two fits (MDPWA and MDF with average solutions) are consistent.

1 Introduction

Here we present the results of fit which is a mass dependent partial wave analysis (MDPWA). MDPWA is carried out not separately for each $\eta\pi^0$ mass bin, but all bins are fitted simultaneously. They are tied together with a mass-dependent function for each partial wave. A likelikhood function depends on not only the angles, but also on mass of $\eta\pi^0$ system.

The advantages of a MDPWA are the next. The results of such analysis don't depend on the ambiguous solutions. So it is not necessary to take an average ambiguous solutions or select between them. We can set the mass dependence forms of wave amplitudes with the same or different parameters. There is the possibility to include in the fit the leakage and find out its intensity. We also can set the mass dependence of background and fit its parameters or to fix backfroud.

2 Analysis

MDPWA of the $\eta\pi^0$ system is carried out for $1.10 < m(\eta\pi^0) < 1.74$ GeV/c^2 and $0 < |t'| < 1.0$ (GeV/c)^2 with and without the leakage. We did also the PWA in each mass bin with $\Delta m = 40$ MeV/c^2 for comparison with the results of MDPWA. We use 7 amplitudes: S_0, P_0, P_-, D_0, D_- (Unnaturally Parity Exchange Waves (UNPE)) and P_+, D_+ (Naturally Parity Exchange Waves(NPE)). Then we are doing the MDPWA.

In MDPWA the extended maximum likelihood function is generalized to include mass dependence

$$\ln\mathcal{L} \propto \sum_i^n \ln I(\Omega_i, m_i) - \int d\Omega dm\ \eta(\Omega, m)\, I(\Omega, m). \quad (1)$$

The angular distribution of $\eta\pi^0$ system is

$$\begin{aligned} I(m,\theta,\varphi) = & \frac{1}{4\pi}\{\Big|S_0(m) + \sqrt{3}P_0(m)d^1_{00}(\theta) + \sqrt{5}D_0(m)d^2_{00}(\theta) \\ & +[\sqrt{6}P_-(m)d^1_{10}(\theta) + \sqrt{10}D_-(m)d^2_{10}(\theta)]\cos\varphi\Big|^2 \\ & +\Big|[\sqrt{6}P_+(m)d^1_{10}(\theta) + \sqrt{10}D_+(m)d^2_{10}(\theta)]\sin\varphi\Big|^2 \\ & +LK(m,\theta,\varphi)\}\ q(m) \\ & +BG(m). \end{aligned} \quad (2)$$

We are doing the MDPWA with the definite form of amplitude wave mass dependence (see [1]). The the $P_+(m)$ may be a resonant or nonresonat version of P_+ wave.

The $LK(m,\theta,\varphi)$ is a leakage of P_+ wave from D_+ wave. Our study of leakage by Monte Carlo simulation of E852 resolution [1] has shown that the relative phase between P_+ wave and the leakage is close to 90°. So we include the leakage incoherently. $q(m)$ is the break-up momentum.

$BG(m)$ is the smooth and isotropic background, which is calculated with the help of the side bands under the η meson signal (see [2]) and fixed in our fits.

Here we use the next forms of mass dependence:

$$
\begin{aligned}
P_+^{(res)}(m) &= a_1\Delta(m, m_1^0, \Gamma_1^0)B_1(q)e^{i\ \alpha_1}; & (3)\\
D_+(m) &= a_2\Delta(m, m_2^0, \Gamma_2^0)B_2(q)[1 + b_1(m - m_2^0) + b_2(m - m_2^0)^2]^{1/2}; & (4)\\
S_0(m) &= a_0|\Delta(m, m_0, \Gamma_0)|; & (5)\\
P_0(m) &= a_3(m - m_{th})^2 e^{-b_3(m-m_{th})}e^{i\varphi_3}; & (6)\\
P_-(m) &= a_4(m - m_{th})^2 e^{-b_4(m-m_{th})}e^{i\varphi_4}; & (7)\\
D_0(m) &= a_5\frac{D_+(m)}{a_2}e^{i\varphi_5}; & (8)\\
LK(m, \theta, \varphi) &= \mid P_{lk}(m) \mid^2 [\sqrt{6}d_{10}^1(\theta)\sin\varphi]^2. & (9)\\
& & (10)
\end{aligned}
$$

Here m is the $\eta\pi^0$-mass, $m_{th} = m_{\pi^0} + m_\eta$ is the threshold mass, $B_l(q)$ is the Blatt-Weisskopf barrier factor , φ_3, φ_4, φ_5 are mass independent phases of UNPE waves. All other designations are from [1].

The Breit-Wigner amplitude $\Delta(m, m_k, \Gamma_k)$ is

$$
\Delta(m, m_k, \Gamma_k) = \frac{m_k^0 \cdot \Gamma_k^0}{(m^2 - (m_k^0)^2) + i(m_k^0\Gamma_k(m))} = e^{i\varphi_k(m)}\left|\Delta(m, m_k^0, \Gamma_k^0)\right|. \quad (11)
$$

Here $\varphi_k(m)$ is a BW phase of wave amplitude and α_1 is the relative production phase of these waves. The widths $\Gamma_k(m)$ are well known functions of mass, which are proportional parameter Γ_k^0.

The $P_{lk}(m)$ leakage is proportional to the D_+-wave mass dependence and has its own normalization factor a_{lk}.

$$
P_{lk}(m) = a_{lk} \cdot \frac{D_+(m)}{a_2}. \quad (12)
$$

All points in our figures are the PWA ambiguous solutions. PWA results are here getting by other PWA code of V.L. Korotkikh and a little shift comparing with results of standart E852 PWA code. The lines are the results of MDPWA. The points are not used in MDPWA. They are shown for comparison with MDPWA curves.

3 Results

The results of MDPWA fits are in Table 1. Our estimation from standart PWA woth the average ambiguous solutions [3] gives zero P_+ leakage from D_+ wave. Fit 1 corresponds to the absent of the leakage. We have found all MDPWA parameters of the mass dependences (3) - (8) in the fit 1. A comparison MDPWA result and PWA points are in Fig. 1. The resonant parameters are consistent with parameters of standart PWA in the statistic error limits.

MDPWA allows us to include the leakage into the angular distribution (see (9)). It is more thin analysis then the leakage including in the mass dependent fit [3]. So we fixed the resonant parameters of D_+, P_+ and all UNPE waves and found a leakage contribution (12) with the magnitude $a_{lk} = 20 \pm 3$.

Then we fixed the leakage magnitude and made free parameters of D_+, P_+ waves. The results are in Table 1 (Fit 2) and in Fig. 2. Total intensity and UNPE waves are in Fig. 3.

The leakage contribution is

$$|LK|^2/|P_+|^2 = 18\% \quad |LK|^2/|D_+|^2 = 5\%$$

The contribution of D_0 wave in the parametrization (8) is

$$|D_0|^2/|D_+|^2 = 40\%$$

Table 1. Parameters of MDPWA of $\eta\pi^0$ system.

Fit	1			2		
	without LK			with LK		
P_+^{res}						
a_1	50	$\pm$	1	46.3	$\pm$	1.5
m_1^0, MeV	1283	$\pm$	7	1276	$\pm$	8
Γ_1^0, MeV	382	$\pm$	24	398	$\pm$	27
α_1	1.37	$\pm$	0.04	1.31	$\pm$	0.04
D_+						
a_2	93	$\pm$	2	93.1	$\pm$	1.9
m_2^0,MeV	1326	$\pm$	3	1325	$\pm$	3
Γ_2^0,MeV	112	$\pm$	5	111	$\pm$	5
b_1,GeV^{-1}	0.5	$\pm$	0.7	0.6	$\pm$	0.7
b_2,GeV^{-2}	33	$\pm$	6	33	$\pm$	6
a_{lk}	0.	fixed		20	fixed	
Leakage study						
	fixed	resonant	param			
a_{lk}	20	$\pm$	3			

References

[1] S.U. Chung *et al.*, Phys. Rev. D **60**, 092001 (1999).

[2] V.L. Korotkikh, L.V. Malinina, "Analysis of $\eta\pi^0$ system with the decay $\eta \to \pi^+\pi^-\pi^0$. Part 1", 2005.

[3] V.L. Korotkikh, L.V. Malinina, Mass dependent fit of PWA results for $\eta\pi^0$ system. Part 2. (2005)

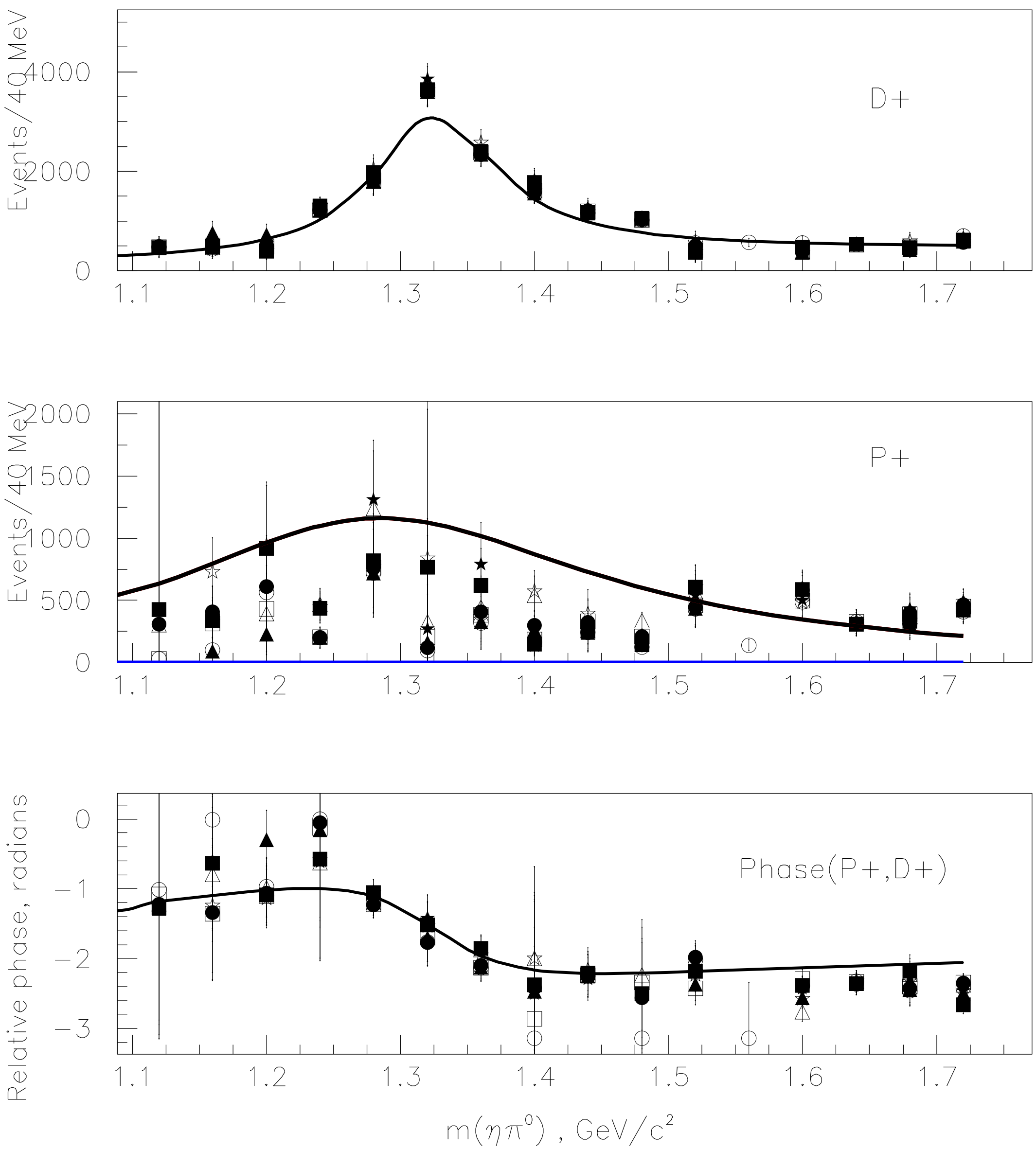

Figure 1: The results of MDPWA for $D+$ and $P+$ waves and phase difference between them. Fit 1 without leakage from D_+ wave. The curves are the results of MDPWA with Brei-Wigner free parameters. The points and errors are from PWA in each mass bin for comparison with MDPWA curves.

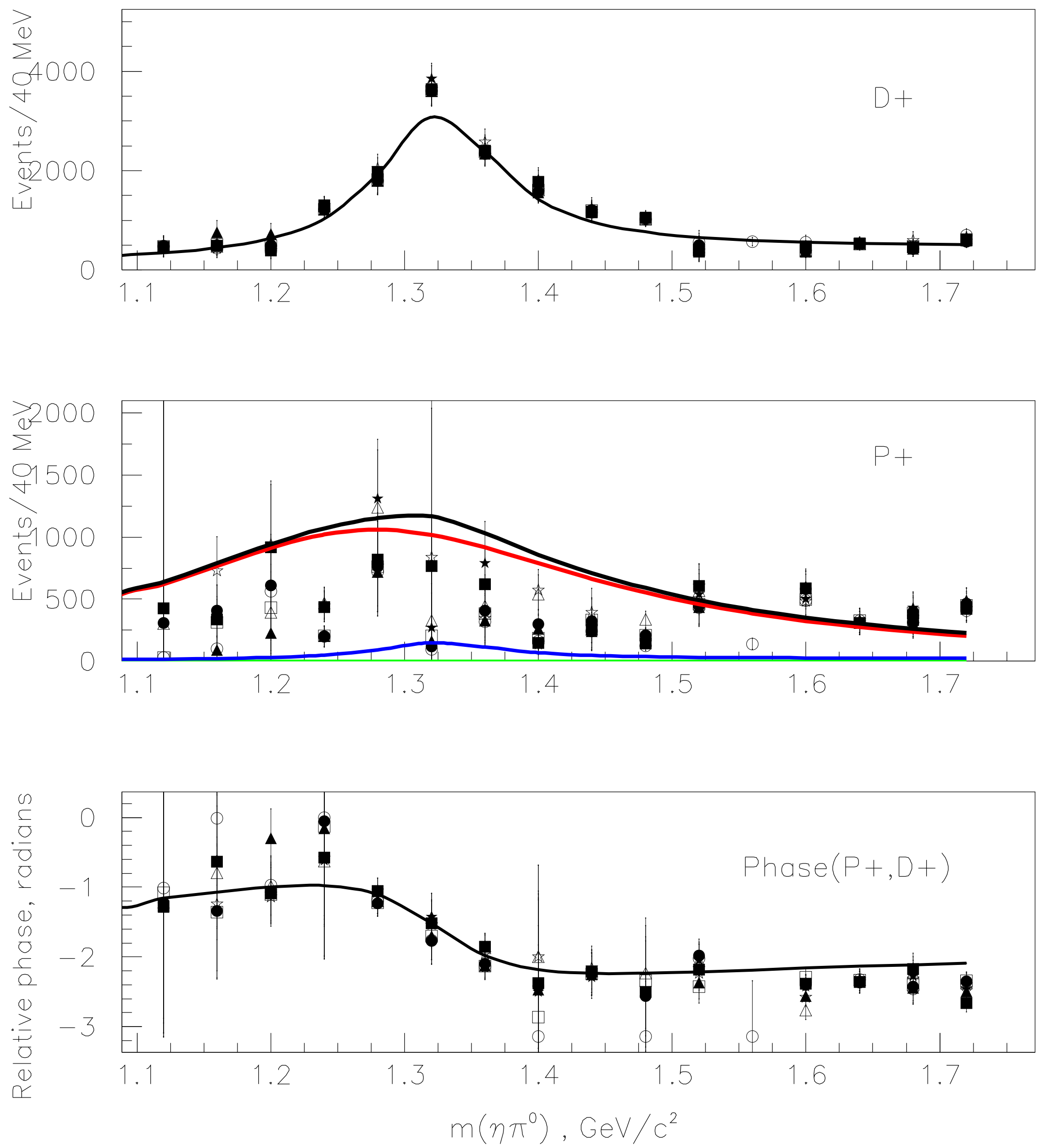

Figure 2: The results of MDPWA for $D+$ and $P+$ waves and phase difference between them. Fit 2, free magnitude of the leakage from D_+ wave.

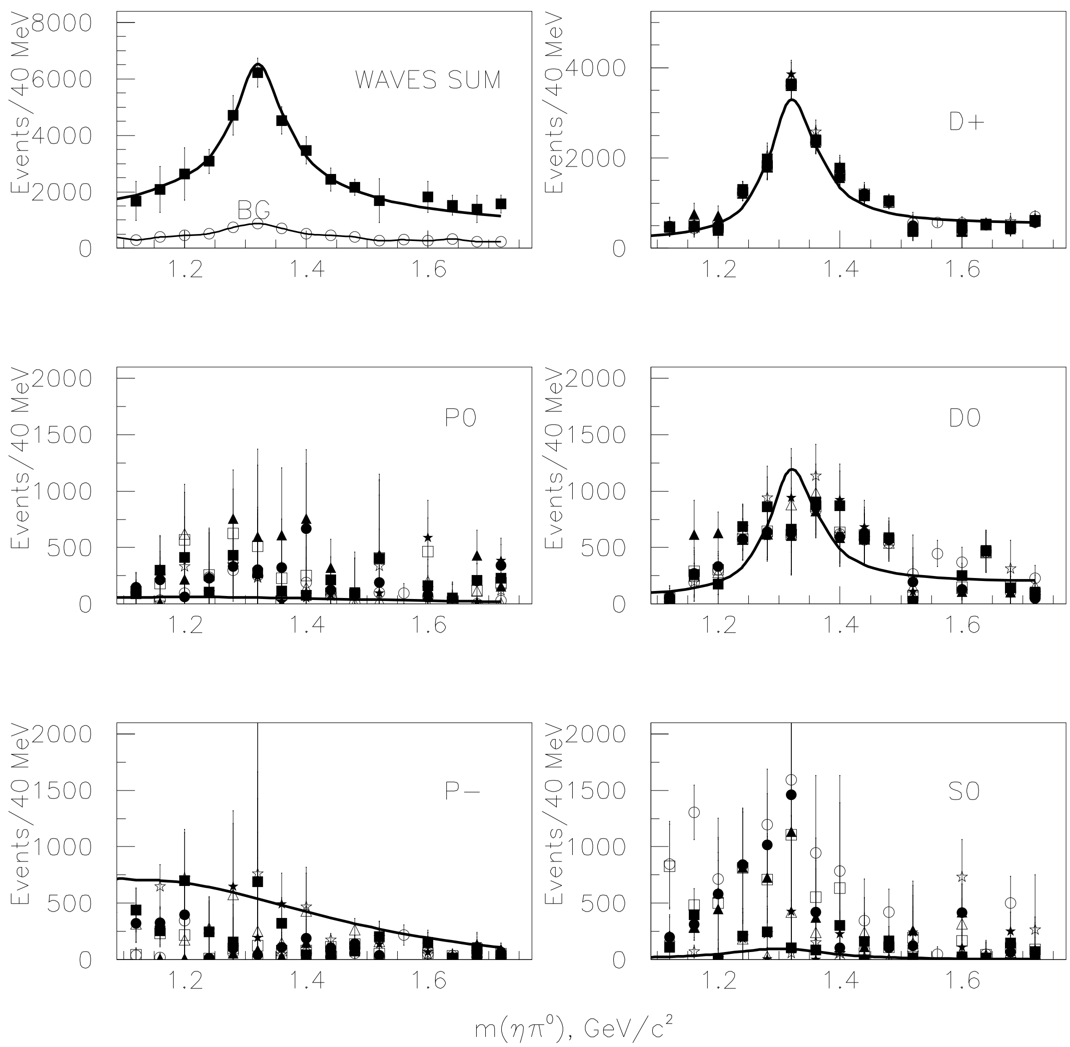

Figure 3: The results of MDPWA for UNPE waves. Fixed leakage from D_+ wave with magnitude from Fit 2.