MDPWA of $\eta\pi^0$ system
Parameters of all waves are free.

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1 Analysis

MDPWA of the $\eta\pi^0$ system is carried out for $1.10 < m(\eta\pi^0) < 1.74$ GeV/c^2 and $0 < |t'| < 1.0$ (GeV/c)^2 with and without the leakage. We did also the PWA in each mass bin with $\Delta m = 40$ MeV/c^2 for comparison with the results of MDPWA. We use 7 amplitudes: S_0, P_0, P_-, D_0, D_- (Unnaturally Parity Exchange Waves (UNPE)) and P_+, D_+ (Naturally Parity Exchange Waves(NPE)).

In MDPWA the extended maximum likelihood function is generalized to include mass dependence

$$\ln\mathcal{L} \propto \sum_i^n \ln I(\Omega_i, m_i) - \int d\Omega dm\ \eta(\Omega, m)\, I(\Omega, m). \tag{1}$$

The angular distribution of $\eta\pi^0$ system is

$$\begin{aligned} I(m,\theta,\varphi) \;=\; & \frac{1}{4\pi}\{\Big|S_0(m) + \sqrt{3}P_0(m)d^1_{00}(\theta) + \sqrt{5}D_0(m)d^2_{00}(\theta) \\ & +[\sqrt{6}P_-(m)d^1_{10}(\theta) + \sqrt{10}D_-(m)d^2_{10}(\theta)]\cos\varphi\Big|^2 \\ & +\Big|[\sqrt{6}P_+(m)d^1_{10}(\theta) + \sqrt{10}D_+(m)d^2_{10}(\theta)]\sin\varphi\Big|^2 \\ & +LK(m,\theta,\varphi)\}\ q(m) \\ & +BG(m). \end{aligned} \tag{2}$$

We are doing the MDPWA with the definite form of amplitude wave mass dependence.

The $LK(m,\theta,\varphi)$ is a leakage of P_+ wave from D_+ wave. Our study of leakage by Monte Carlo simulation of E852 resolution has shown that the relative phase between P_+ wave and the leakage is close to 90°. So we include the leakage incoherently. $q(m)$ is the break-up momentum.

$BG(m)$ is the smooth and isotropic background, which is calculated with the help of the side bands under the η meson signal and fixed in our fits.

Here we use the next forms of mass dependence:

$$\begin{aligned} P_+^{(res)}(m) &= a_1\Delta(m, m_1^0, \Gamma_1^0)B_1(q)e^{i\ \alpha_1}; & (3)\\ D_+(m) &= a_2\Delta(m, m_2^0, \Gamma_2^0)B_2(q)[1 + b_1(m - m_2^0) + b_2(m - m_2^0)^2]^{1/2}; & (4)\\ S_0(m) &= a_0\Delta(m, m_0, \Gamma_0); & (5)\\ LK(m, \theta, \varphi) &= \mid P_{lk}(m) \mid^2 [\sqrt{6}d_{10}^1(\theta)\sin\varphi]^2. & (6)\\ & & (7) \end{aligned}$$

The Breit-Wigner amplitude $\Delta(m, m_k, \Gamma_k)$ is

$$\Delta(m, m_k, \Gamma_k) = \frac{m_k^0 \cdot \Gamma_k^0}{(m^2 - (m_k^0)^2) + i(m_k^0\Gamma_k(m))} = e^{i\phi_k(m)} \left|\Delta(m, m_k^0, \Gamma_k^0)\right|. \quad (8)$$

Here $\phi_k(m)$ is a BW phase of wave amplitude and α_1 is the relative production phase of these waves. The widths $\Gamma_k(m)$ are well known functions of mass, which are proportional parameter Γ_k^0.

Here m is the $\eta\pi^0$-mass, $m_{th} = m_{\pi^0} + m_\eta$ is the threshold mass, $B_l(q)$ is the Blatt-Weisskopf barrier factor.

The leakage $P_{lk}(m)$ is proportional to the D_+-wave mass dependence and has its own normalization factor a_{lk}.

$$P_{lk}(m) = X_{lk} \cdot \frac{D_+(m)}{a_2}. \quad (9)$$

Fit 1. UNPWs are as a polynomial background. Constant phase of wave.

$$\begin{aligned} P_0(m) &= a_3(1 - (m - m_3)^2/(m - m_{th})^2)e^{i\varphi_3}; & (10)\\ P_-(m) &= a_4(1 - (m - m_4)^2/(m - m_{th})^2)e^{i\varphi_4}; & (11)\\ D_0(m) &= a_5(1 - (m - m_5)^2/(m - m_{th})^2)e^{i\varphi_5}; & (12)\\ D_-(m) &= a_6(1 - (m - m_6)^2/(m - m_{th})^2)e^{i\varphi_6}; & (13)\\ & & (14) \end{aligned}$$

If a wave is negative we set it to zero. The intensity maximum of these wave is equal to $m = m_k$. φ_k are mass independent phases of UNPE waves except S_0 wave.

Fit 2. All waves BW resonances.

$$
\begin{aligned}
P_0(m) &= a_3\Delta(m, m_1^0, \Gamma_1^0)B_1(q); &(15)\\
P_-(m) &= a_4\Delta(m, m_1^0, \Gamma_1^0)B_1(q); &(16)\\
D_0(m) &= a_5\Delta(m, m_2^0, \Gamma_2^0)B_2(q)[1 + b_1(m - m_2^0) + b_2(m - m_2^0)^2]^{1/2}; &(17)\\
D_-(m) &= a_6\Delta(m, m_2^0, \Gamma_2^0)B_2(q)[1 + b_1(m - m_2^0) + b_2(m - m_2^0)^2]^{1/2};; &(18)\\
& &(19)
\end{aligned}
$$

Fit 3. D_0 is as D_+ and other UNPW are polynomial background.

$$
\begin{aligned}
P_0(m) &= a_3(1 - (m - m_3)^2/(m - m_{th})^2)e^{i\varphi_3}; &(20)\\
P_-(m) &= a_4(1 - (m - m_4)^2/(m - m_{th})^2)e^{i\varphi_4}; &(21)\\
D_0(m) &= a_5\frac{D_+(m)}{a_2}e^{i\varphi_5}; &(22)\\
D_-(m) &= a_6(1 - (m - m_6)^2/(m - m_{th})^2)e^{i\varphi_6}; &(23)\\
& &(24)
\end{aligned}
$$

Fit 4.Smooth UNPW background.

$$
\begin{aligned}
P_0(m) &= a_3(m - m_{th})^2e^{-b_3(m-m_{th})}e^{i\varphi_3}; &(25)\\
P_-(m) &= a_4(m - m_{th})^2e^{-b_4(m-m_{th})}e^{i\varphi_4}; &(26)\\
D_0(m) &= a_5\frac{D_+(m)}{a_2}e^{i\varphi_5}; &(27)\\
D_-(m) &= a_6(m - m_{th})e^{-b_6(m-m_{th})}e^{i\varphi_6}; &(28)\\
& &(29)
\end{aligned}
$$

The intensity maximum of these wave is equal to $m = 2/b_k$.

Fit 5.Other form of UNPW background.

$$
\begin{aligned}
P_0(m) &= a_3(m - m_{th})(1 + b_3(m - m_{th}))e^{i\varphi_3}; &(30)\\
P_-(m) &= a_4(m - m_{th})(1 + b_4(m - m_{th}))e^{i\varphi_4}; &(31)\\
D_0(m) &= a_5\frac{D_+(m)}{a_2}e^{i\varphi_5}; &(32)\\
D_-(m) &= a_6(m - m_{th})(1 + b_6(m - m_{th}))e^{i\varphi_4}; &(33)\\
& &(34)
\end{aligned}
$$

All points with errors in our figures are the PWA ambiguous solutions. The lines are the results of MDPWA. The points are not used in MDPWA. They are shown for comparison with MDPWA curves.

2 Results

Below we give the fit parameters and figures for five fits.

FIT1

This fit has a maximum free parameters of MDPWA. The parameters of $a_0(980)$ are taken from fit of our data in the mass region of $a_0(980)$ and fixed.

— bump of P0,P-,D0,D-

— free 1-22,26 par

parameter value error

1'p+ ' 0.25327E+02 0.13975E+01

2'm1 ' 0.12564E+01 0.10280E-01

3'g10 ' 0.31931E+00 0.34287E-01

4'ph10 ' 0.10586E+01 0.53263E-01

5'ph11 ' 0.00000E+00 0.00000E+00 — not used

6'd+ ' 0.28117E+02 0.60474E+00

7'm2 ' 0.13265E+01 0.29526E-02

8'g20 ' 0.10398E+00 0.50806E-02

9'bg1 ' 0.32558E+00 0.93367E+00

10'bg2 ' 0.34643E+02 0.63754E+01

11'p0 ' 0.21897E+02 0.13618E+01

12'ph20 ' 0.30431E+01 0.11014E+00

13'mp0 ' 0.13129E+01 0.34730E-01

14'p- ' 0.20253E+02 0.11105E+01

15'ph30 ' 0.13215E+04 0.90914E-01

16'mp- ' 0.10770E+01 0.99417E-02

17'd0 ' 0.23211E+02 0.90193E+00

18'ph40 ' -0.19712E+01 0.97301E-01

19'md0 ' 0.14541E+01 0.48195E-01

20'd- ' 0.12123E+02 0.16245E+01

21'ph50 ' -0.15412E+02 0.10141E+00

22'md- ' 0.95206E+00 0.97795E-02

23'a01 ' 0.32960E+02 0.00000E+00 — fix

24'a0m ' 0.99751E+00 0.00000E+00 — fix

25'a0g ' 0.10222E+00 0.00000E+00 — fix

26'xleak ' 0.51068E+01 0.32342E+00

FIT2

In the Fit2 - Fit5 we fix the parameters of $a_2(1320)$ and $a_0(980)$.
1)P0,P- as P+, D0,D- as D+
— fix a0, a2
— free 1-5,6,11-22,26 par
parameter value error
1'p+ ' 0.25018E+02 0.61843E+00
2'm1 ' 0.12058E+01 0.94756E-02
3'g10 ' 0.72184E+00 0.24271E-01
4'ph10 ' 0.96971E+00 0.49561E-01
5'ph11 ' 0.00000E+00 0.00000E+00 — not used
6'd+ ' 0.29720E+02 0.39951E+00
7'm2 ' 0.13265E+01 0.00000E+00 — fix
8'g20 ' 0.10398E+00 0.00000E+00 — fix
9'bg1 ' 0.32558E+00 0.00000E+00 — fix
10'bg2 ' 0.34643E+02 0.00000E+00 — fix
11'p0 ' 0.25052E+02 0.82532E+00
12'ph20 ' 0.67412E+00 0.10858E+00
13'mp0 ' 0.13269E+01 0.10000E+01
14'p- ' 0.22067E+02 0.99362E+00
15'ph30 ' 0.13218E+04 0.93367E-01
16'mp- ' 0.10774E+01 0.10000E+01
17'd0 ' 0.21351E+02 0.58822E+00
18'ph40 ' -0.22707E+01 0.10028E+00
19'md0 ' 0.14660E+01 0.10000E+01
20'd- ' 0.38561E+01 0.76802E+00
21'ph50 ' -0.15322E+02 0.24289E+00
22'md- ' 0.95161E+00 0.10000E+01
23'a01 ' 0.32960E+02 0.00000E+00 — fix
24'a0m ' 0.99751E+00 0.00000E+00 — fix
25'a0g ' 0.10222E+00 0.00000E+00 — fix
26'xleak ' 0.15032E-09 0.32061E+00

Fit3
2)P0,P-,D- –¿ bumps , D0 as D+
— fix a0,a2
— free 1-5,6,11-22,26 par
parameter value error
1'p+ ' 0.28014E+02 0.12269E+01
2'm1 ' 0.12860E+01 0.10679E-01

3'g10 ' 0.53189E+00 0.46226E-01
4'ph10 ' 0.14991E+01 0.52143E-01
5'ph11 ' 0.00000E+00 0.00000E+00 — not used
6'd+ ' 0.28269E+02 0.40876E+00
7'm2 ' 0.13139E+01 0.00000E+00 — fix
8'g20 ' 0.11182E+00 0.00000E+00 — fix
9'bg1 ' 0.37432E+01 0.00000E+00 — fix
10'bg2 ' 0.20588E+02 0.00000E+00 — fix
11'p0 ' 0.19918E+02 0.83438E+00
12'ph20 ' 0.13211E+04 0.53684E-01
13'mp0 ' 0.10476E+01 0.93191E-02
14'p- ' 0.14104E+02 0.10746E+01
15'ph30 ' 0.13210E+04 0.67465E-01
16'mp- ' 0.10435E+01 0.39318E-02
17'd0 ' 0.18859E+02 0.57665E+00
18'ph40 ' 0.24411E+01 0.00000E+00
19'md0 ' 0.14541E+01 0.00000E+00
20'd- ' 0.16480E+02 0.84016E+00
21'ph50 ' -0.15648E+02 0.50804E-01
22'md- ' 0.16376E+01 0.13499E+00
23'a01 ' 0.40341E+02 0.00000E+00 — fix
24'a0m ' 0.99751E+00 0.00000E+00 — fix
25'a0g ' 0.14794E+00 0.00000E+00 — fix
26'xleak ' 0.10168E+01 0.94030E+00

FIT4
3)P0,P-, D- as smooth bcgr, D0 as D+
— fix a0,a2
— free 1-5,6,11-22,26 par
parameter value error
1'p+ ' 0.28664E+02 0.13631E+01
2'm1 ' 0.12746E+01 0.66565E-02
3'g10 ' 0.42821E+00 0.28763E-01
4'ph10 ' 0.14734E+01 0.38009E-01
5'ph11 ' 0.00000E+00 0.00000E+00 — not used
6'd+ ' 0.27121E+02 0.43323E+00
7'm2 ' 0.13139E+01 0.10000E+00 — fix
8'g20 ' 0.11182E+00 0.10000E+00 — fix
9'bg1 ' 0.37432E+01 0.10000E+00 — fix

10'bg2 ' 0.20588E+02 0.10000E+00 — fix
11'p0 ' 0.94811E+03 0.72324E+02
12'ph20 ' 0.13558E+01 0.51117E-01
13'mp0 ' 0.47482E+01 0.16709E+00
14'p- ' 0.87404E+02 0.13107E+02
15'ph30 ' -0.19241E+00 0.80865E-01
16'mp- ' 0.16330E+01 0.16641E+00
17'd0 ' 0.13896E+02 0.70570E+00
18'ph40 ' 0.24411E+01 0.00000E+00
19'md0 ' 0.14541E+01 0.00000E+00
20'd- ' 0.96012E+04 0.19199E+04
21'ph50 ' 0.41336E+01 0.60134E-01
22'md- ' 0.15196E+02 0.12635E+01
23'a01 ' 0.51253E+02 0.00000E+00 — fix
24'a0m ' 0.99751E+00 0.00000E+00 — fix
25'a0g ' 0.10222E+00 0.00000E+00 — fix
26'xleak ' 0.29393E+01 0.93074E+00

FIT5
4)P0,P-, D- as m*(1+b*m), D0 as D+
— at fix a2,a0
— free 1-5,6,11-22,26 par
par30-e852-all.par-res4
parameter value error
1'p+ ' 0.23504E+02 0.12900E+01
2'm1 ' 0.12903E+01 0.17665E-01
3'g10 ' 0.67054E+00 0.77870E-01
4'ph10 ' 0.16083E+01 0.73022E-01
5'ph11 ' 0.00000E+00 0.00000E+00 — not used
6'd+ ' 0.28701E+02 0.39645E+00
7'm2 ' 0.13139E+01 0.00000E+00 — fix
8'g20 ' 0.11182E+00 0.00000E+00 — fix
9'bg1 ' 0.37432E+01 0.00000E+00 — fix
10'bg2 ' 0.20588E+02 0.00000E+00 — fix
11'p0 ' 0.12750E+03 0.79318E+01
12'ph20 ' 0.14670E+01 0.57445E-01
13'mp0 ' -0.13705E+01 0.46179E-01
14'p- ' 0.17197E+02 0.82144E+01
15'ph30 ' -0.88265E-01 0.90876E-01

16'mp- ' -0.92073E-01 0.49338E+00
17'd0 ' 0.18759E+02 0.80583E+00
18'ph40 ' 0.24411E+01 0.00000E+00
19'md0 ' 0.14541E+01 0.00000E+00
20'd- ' 0.90454E+02 0.15615E+02
21'ph50 ' 0.40649E+01 0.13377E+00
22'md- ' -0.20979E+01 0.11126E+00
23'a01 ' 0.37305E+02 0.00000E+00 — fix
24'a0m ' 0.10075E+01 0.00000E+00 — fix
25'a0g ' 0.18865E+00 0.00000E+00 — fix
26'xleak ' 0.16432E+01 0.11062E+01

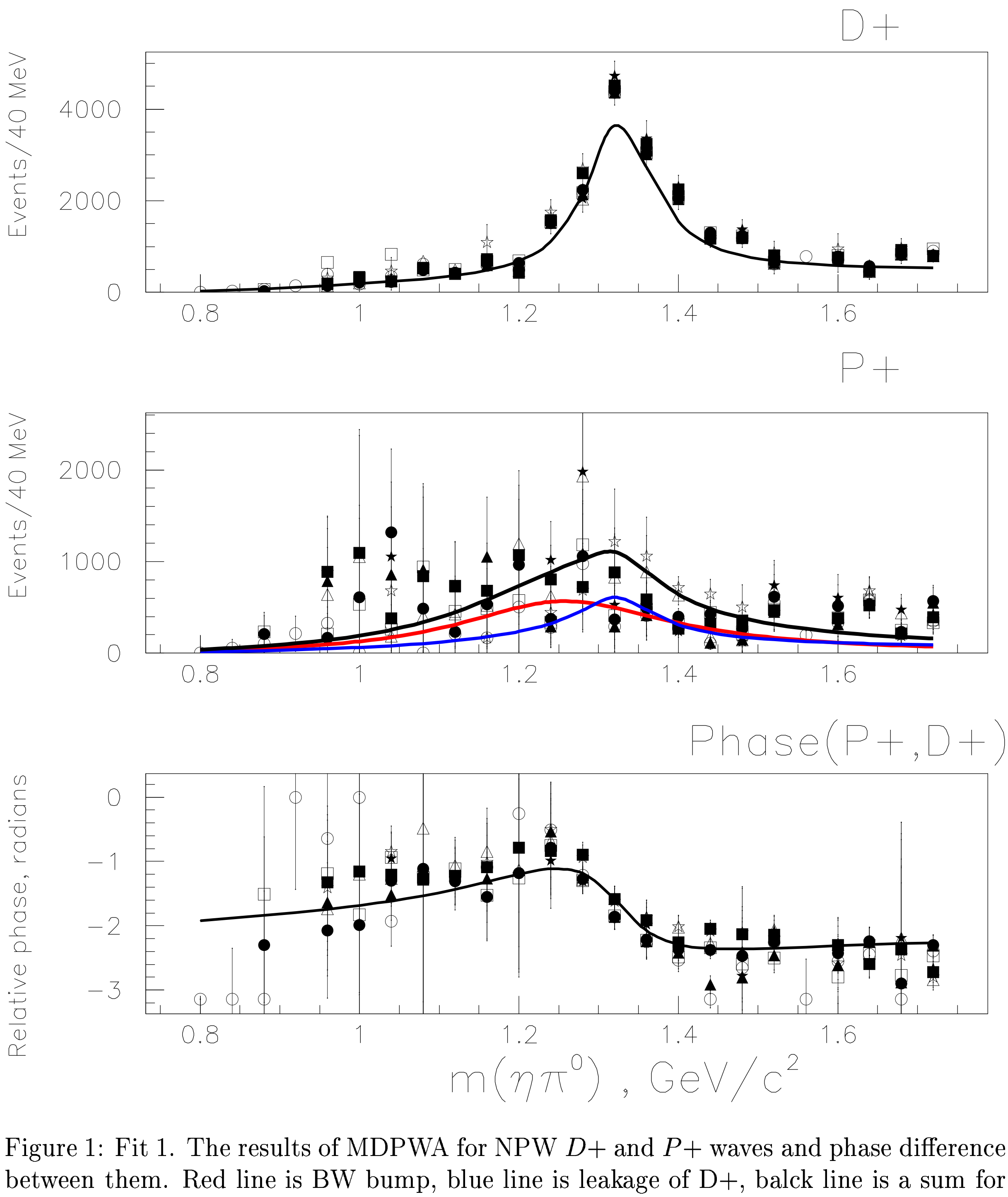

Figure 1: Fit 1. The results of MDPWA for NPW $D+$ and $P+$ waves and phase difference between them. Red line is BW bump, blue line is leakage of D+, balck line is a sum for P+ intensity.

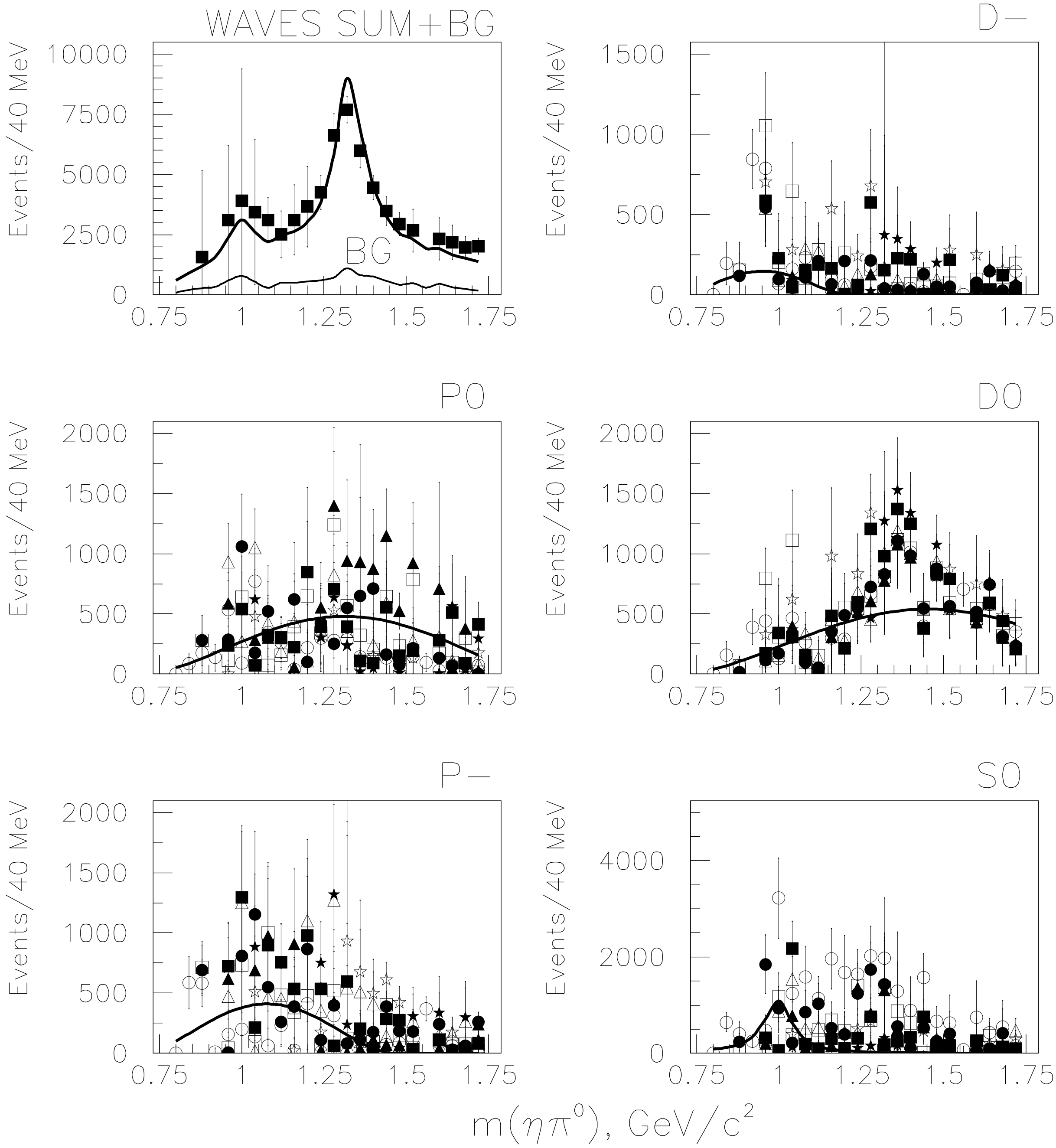

Figure 2: Fit 1. The results of MDPWA for UNPW and a sum of waves.

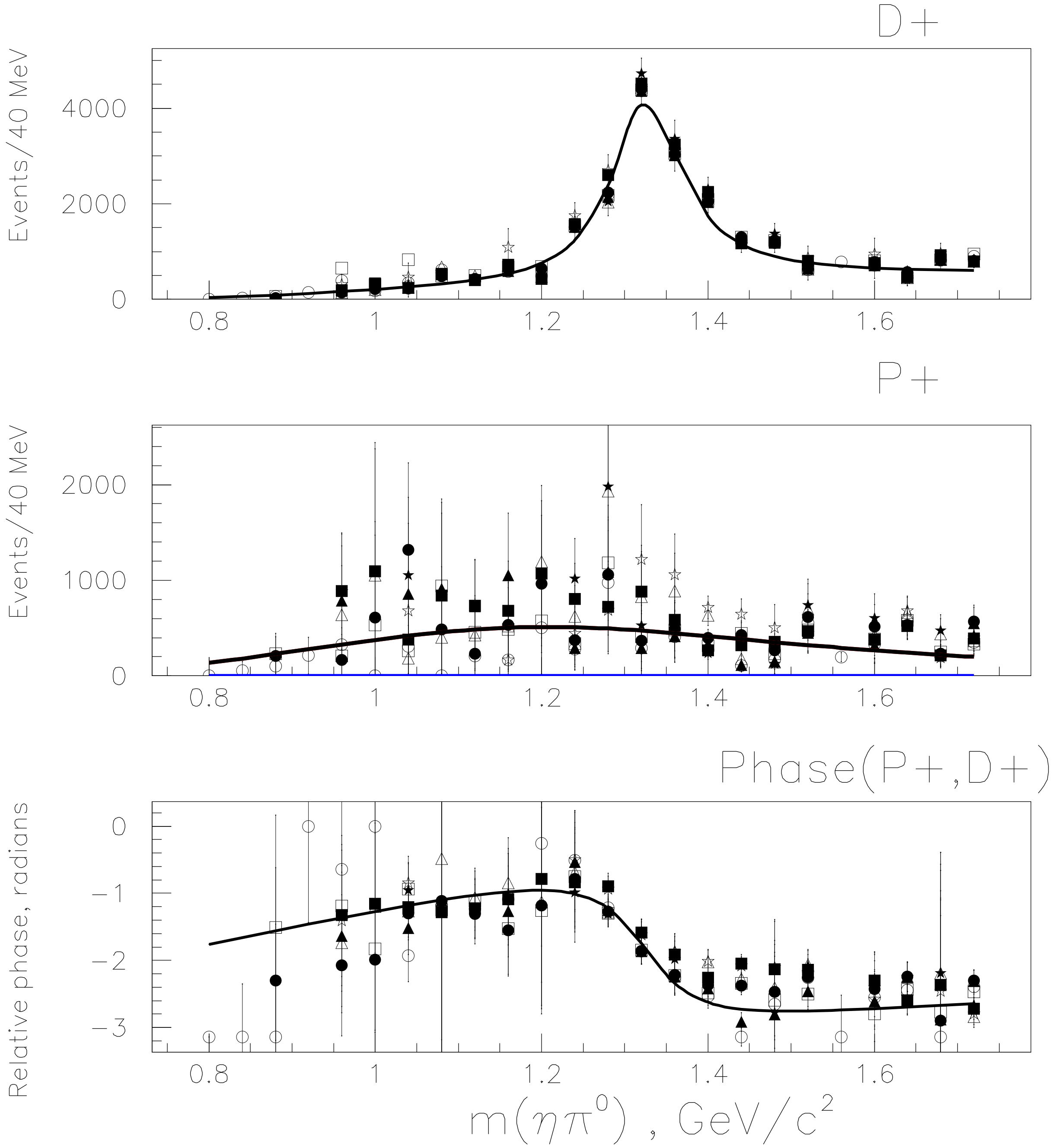

Figure 3: Fit 2. The results of MDPWA for NPW $D+$ and $P+$ waves and phase difference between them.

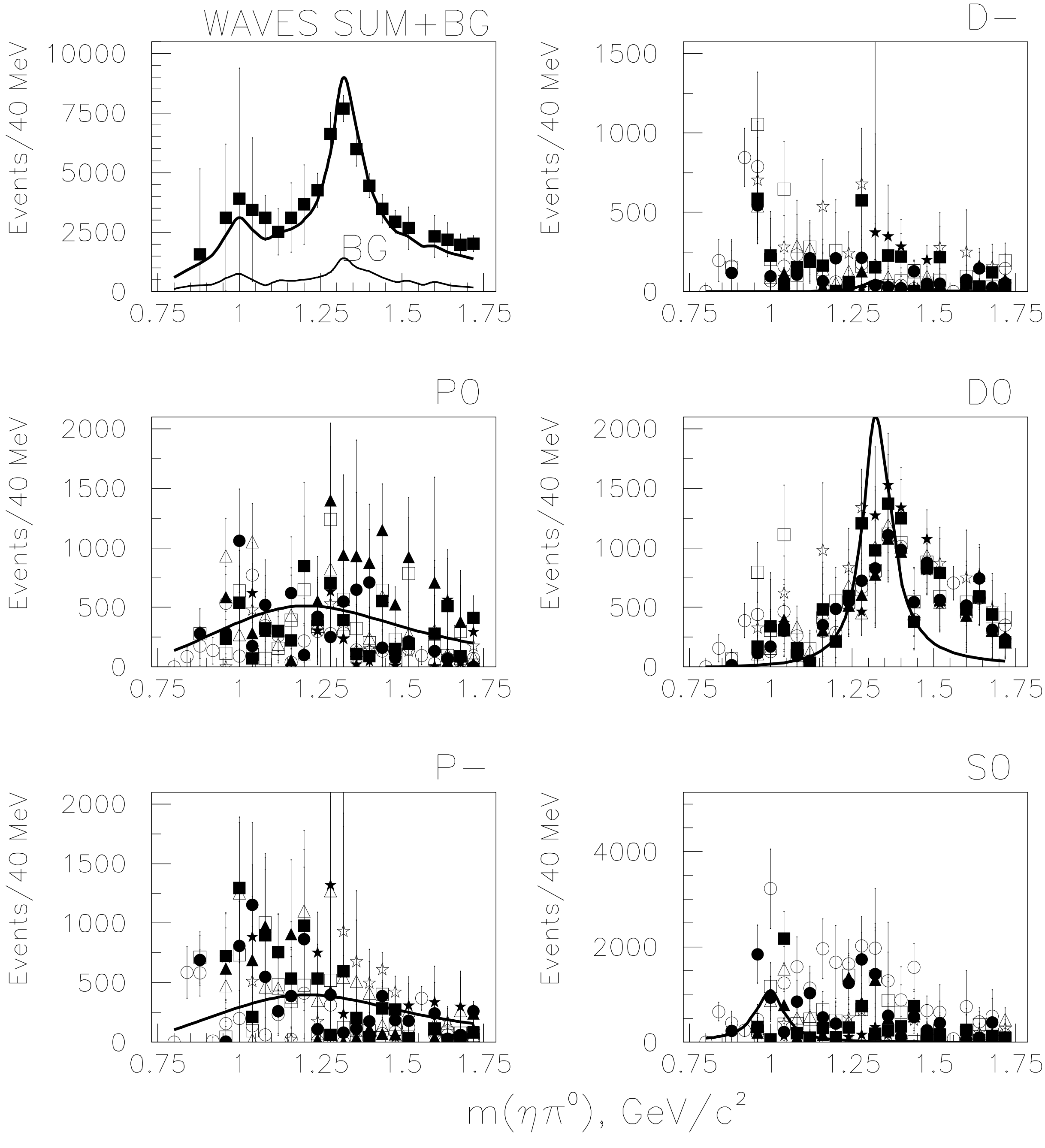

Figure 4: Fit 2. The results of MDPWA for UNPW and a sum of waves.

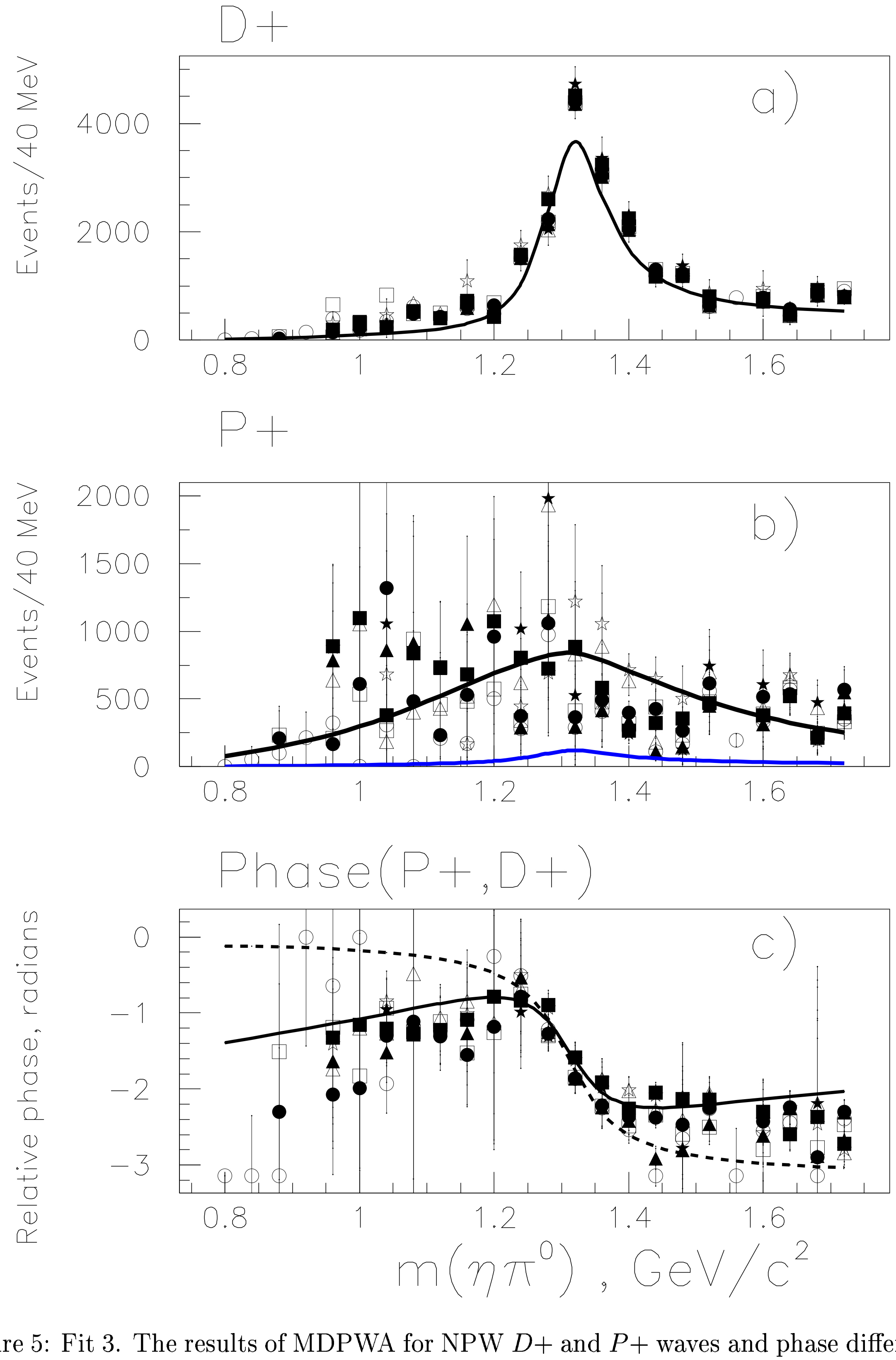

Figure 5: Fit 3. The results of MDPWA for NPW $D+$ and $P+$ waves and phase difference between them. a)D_+ wave intensity, b)P_+ wave intensity and a leakage contribution, c)the relative phase ($P_+ - D_+ +$ constant production phase), dotted line is only resonant D_+ phase.

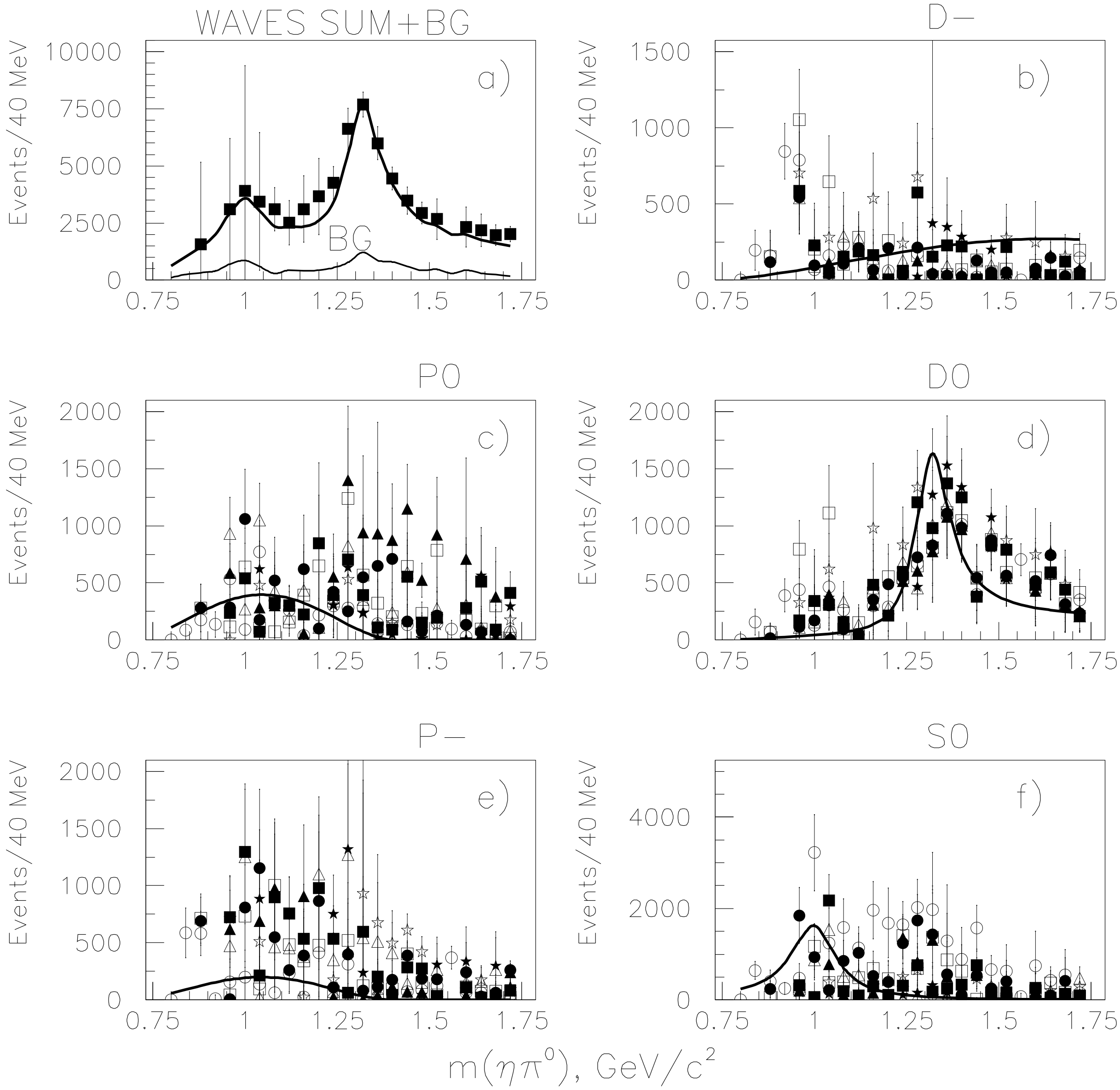

Figure 6: Fit 3. The results of MDPWA for UNPW. a) A sum of waves and background, b)D_- wave intensity, c)P_0 wave intensity, d)D_0 wave intensity, which was fitted with fixed BW resonant parameters as for D_+ wave, e)P_- wave intensity, f)S_0 wave intensity with fixed mass parameter of $a_0(980)$. The waves P_0, P_-, D_- were fitted as polinomial-exponential background with constant phase.

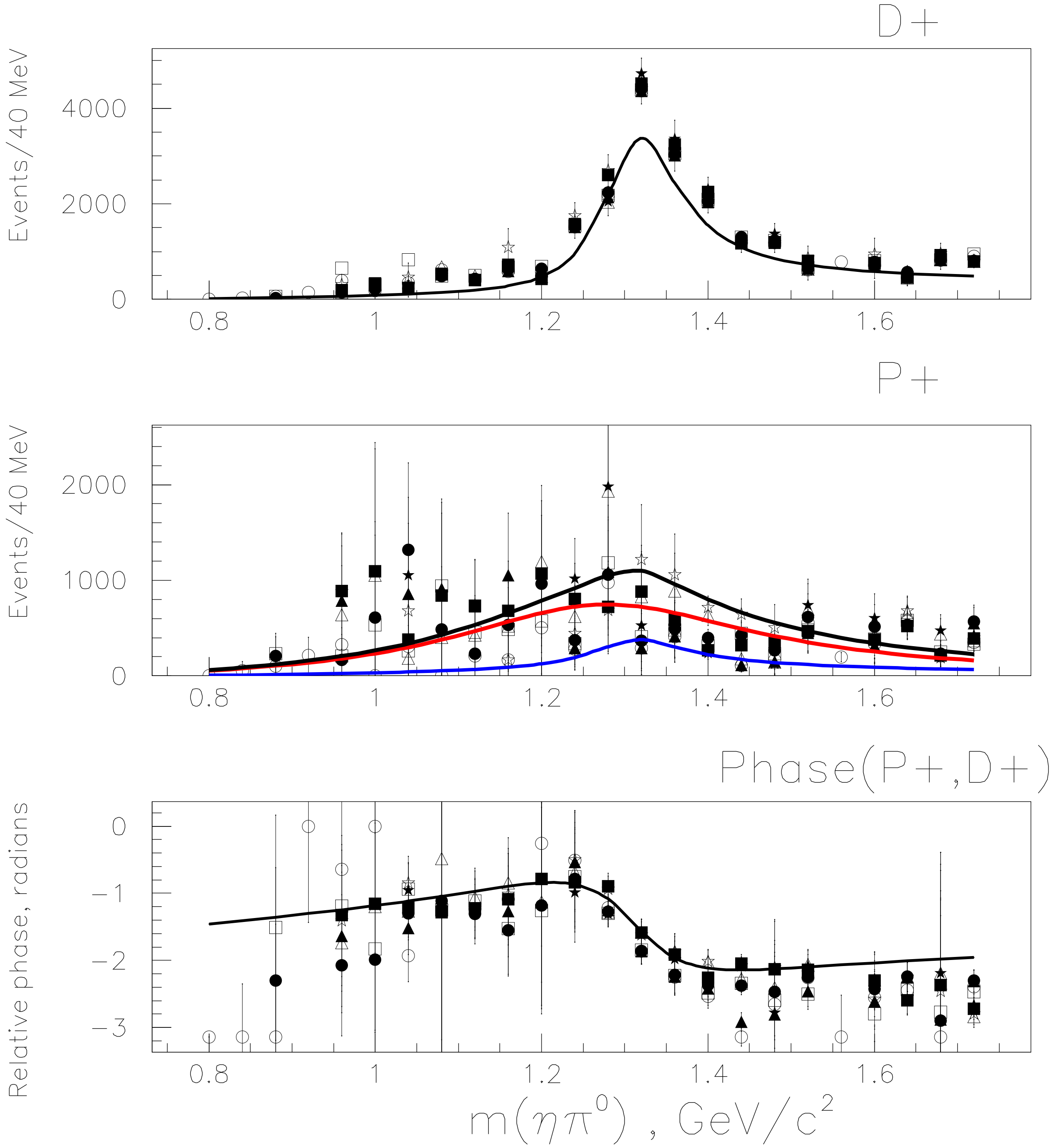

Figure 7: Fit 4. The results of MDPWA for NPW $D+$ and $P+$ waves and phase difference between them.

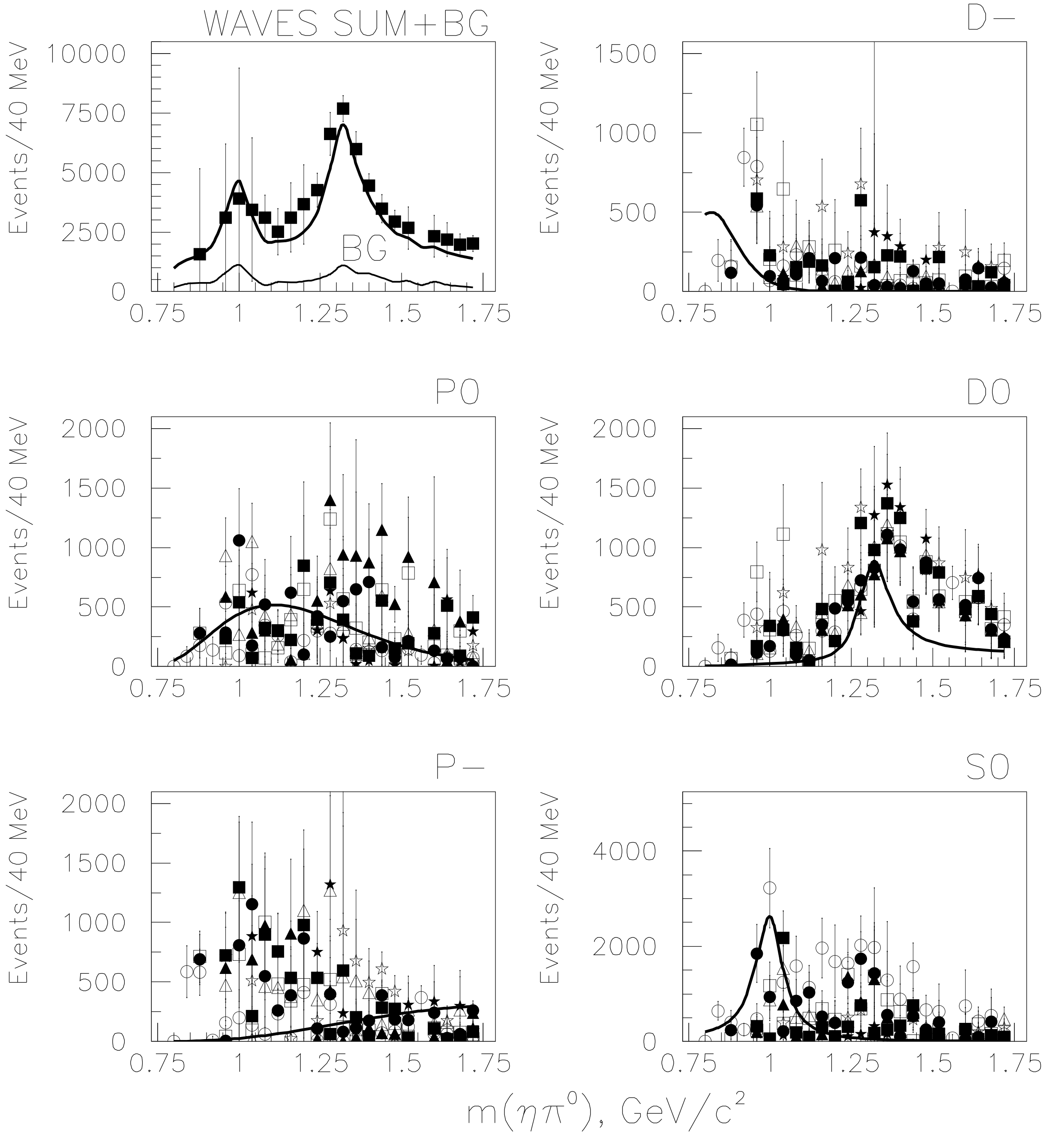

Figure 8: Fit 4. a)The results of MDPWA for UNPW and a sum of waves.

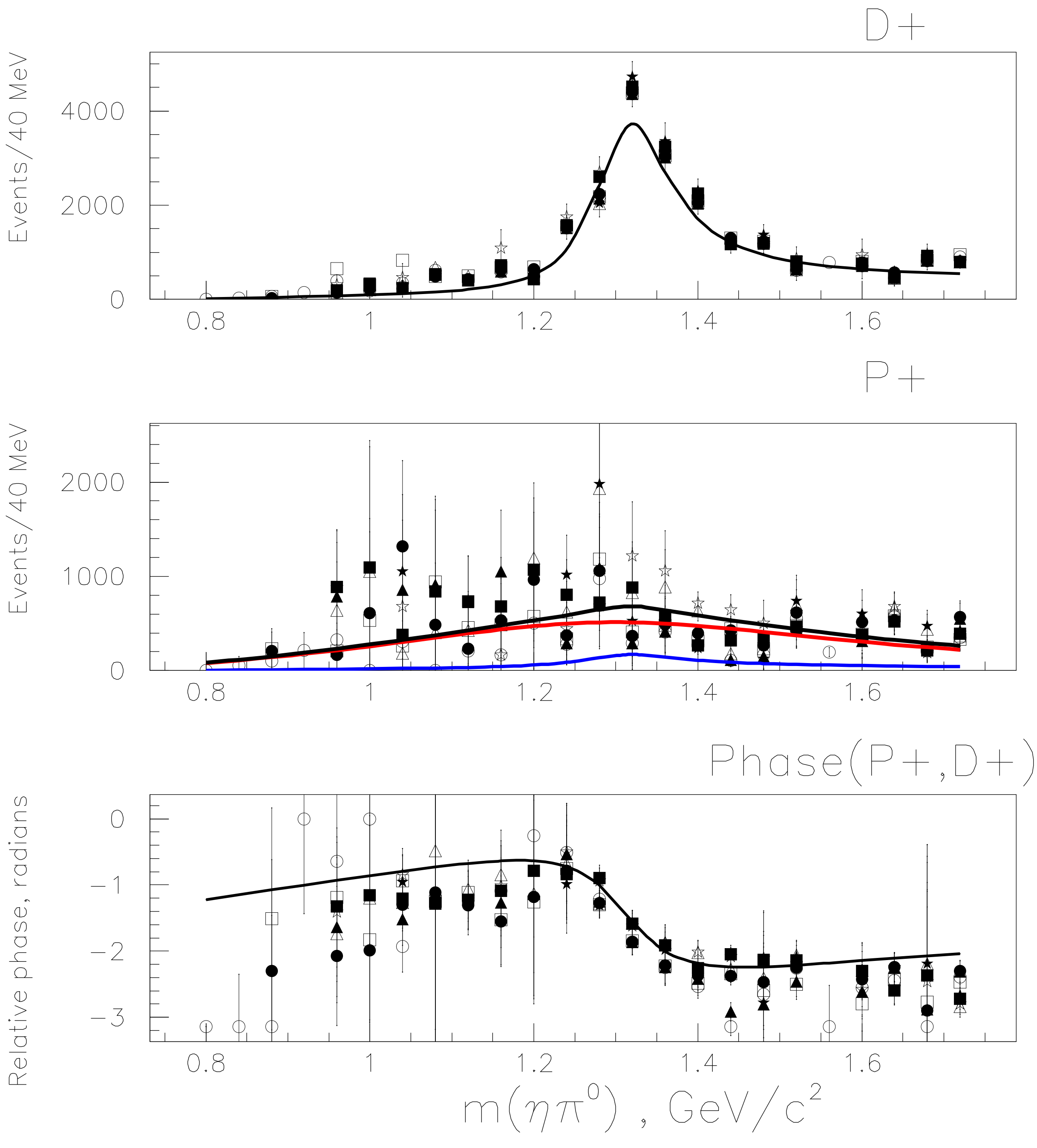

Figure 9: Fit 5. The results of MDPWA for NPW $D+$ and $P+$ waves and phase difference between them.

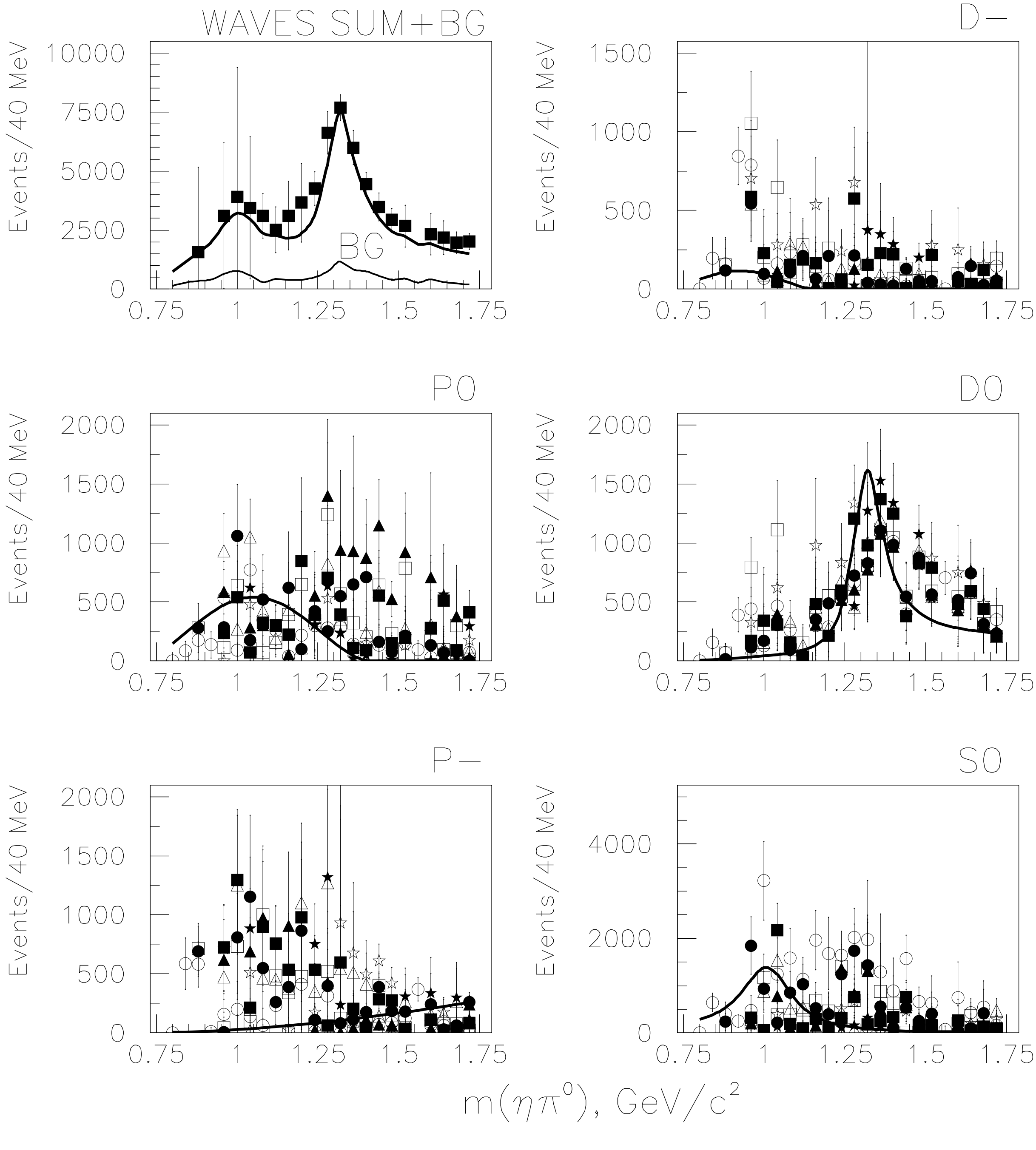

Figure 10: Fit 5. The results of MDPWA for UNPW and a sum of waves.