

# Confirmation of a $\pi_1^0$ Exotic Meson in the $\eta\pi^0$ System

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The exclusive reaction  $\pi^- p \rightarrow \eta\pi^0 n$  (where  $\eta \rightarrow \pi^+\pi^-\pi^0$ ) at 18 GeV/c has been studied in Brookhaven experiment E852. A partial wave analysis has been performed on a sample of 23 492  $\eta\pi^0 n$  events. The results of a mass-dependent fit are consistent with a resonant hypothesis for the  $P_+$  wave, thus providing evidence for a neutral exotic meson. This meson has  $J^{PC} = 1^{-+}$ , a mass of  $1257 \pm 20 \pm 25$  MeV/c<sup>2</sup>, and a width of  $354 \pm 64 \pm 60$  MeV/c<sup>2</sup>.

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## INTRODUCTION.

Exotic mesons with quantum numbers  $J^{PC} = 0^{--}, 1^{-+}, 2^{+-}, \dots$  do not mix with quark-antiquark mesons and thus offer a natural testing ground for QCD. Exotic mesons have been discussed [1–10] for many years but have only recently been observed experimentally. The underlying structure of the negatively charged exotic state with  $J^{PC} = 1^{-+}$  observed in this experiment [11, 12] at 1400 MeV decaying into  $\eta\pi^-$  is not yet understood.

Study of the resonant structure of the neutral  $\eta\pi^0$  system near 1400 MeV can be very important in attempting to understand this underlying structure. An important characteristic of the  $\eta\pi^0$  system, unlike the  $\eta\pi^-$  system, is that  $C$ -parity is a good quantum number. The other distinguishing feature is that the production mechanism for the charge exchange reaction  $\pi^- p \rightarrow \eta\pi^0 n$  cannot involve the exchange of an isospin  $I = 0$  system and thus pomeron exchange is ruled out. These characteristics make the  $\eta\pi^0$  system an excellent one to clarify the properties of this exotic state.

The Crystal Barrel experiment [13] confirmed the existence of resonant structure in the  $\eta\pi^-$  system using stopped antiprotons in liquid deuterium in the reaction  $\bar{p}n \rightarrow \pi^-\pi^0\eta$ . Later this group analyzed data on  $\bar{p}p$

annihilation at rest into  $\pi^0\pi^0\eta$  [14] and presented evidence for an exotic  $1^{-+}$  resonance in the  $\eta\pi^0$  system with  $M = (1360 \pm 25)$  MeV/c<sup>2</sup> and  $\Gamma = (220 \pm 90)$  MeV/c<sup>2</sup>.

The  $\eta\pi^0$  state has been studied in the GAMS experiment [15] using the reaction  $\pi^- p \rightarrow \eta\pi^0 n$ ,  $\eta \rightarrow 2\gamma$ ,  $\pi^0 \rightarrow 2\gamma$  at 32, 38 and 100 GeV/c. They showed that the intensity of the  $P_+$  wave has a wide bump at  $M = 1300$  MeV/c<sup>2</sup>. This structure was difficult to characterize because of the presence of ambiguities in the amplitude analysis. However, the statistics of the 38 GeV/c data was sufficient so that the method of Sadovsky [16] could be used to resolve the ambiguity, and they were able to present evidence for the  $\pi_1(1400)$  exotic state.

The VES experiment also observed a peak in the  $P_+$  wave of the  $\eta\pi^0$  system near 1400 MeV/c<sup>2</sup> [17]. In their most recent publication [18], using theoretical arguments the authors state that the peak can be understood without requiring an exotic meson.

An analysis of E852 data using the reaction  $\pi^- p \rightarrow \eta\pi^0 p$  (with  $\eta \rightarrow 2\gamma$ ) was recently reported [19]. A bump in the  $P_+$  wave of the  $\eta\pi^0$  system was observed at  $M(\eta\pi^0) = 1272$  MeV/c<sup>2</sup> with a large width ( $\Gamma = 660$  MeV/c<sup>2</sup>) when fitting all the data using the method of Ref. [12]. For small  $t'$ , they observe a width of 190 MeV/c<sup>2</sup>. The authors chose not to claim evidence for exotic  $\pi_1(1400)$  meson production.

In the present analysis we have studied the reaction  $\pi^-p \rightarrow \eta\pi^0n$  at 18 GeV/c in E852, using the charged  $\eta \rightarrow \pi^+\pi^-\pi^0$  decay. The advantage of this mode over the all-neutral final state is that the production vertex point is defined by charged tracks. This improves the mass resolution as well as the ability to require that the interaction took place within the liquid hydrogen target.

## EXPERIMENTAL SETUP AND DATA SELECTION

The data for this analysis was obtained at the Alternating Gradient Synchrotron (BNL USA). Using an 18 GeV/c  $\pi^-$  beam interacting in a liquid hydrogen target, a total of 750 million triggers were acquired of which 108 million were of a type designed to enrich the exclusive final state  $\pi^-p \rightarrow \pi^+\pi^-4\gamma n$ . A total of 6 million events of this type were fully reconstructed. The data were kinematically fit [20] to select events consistent with the  $\pi^-\pi^+\pi^0\pi^0n$  hypothesis (with a confidence level of at least 0.01%) yielding about 4 million events. Of those, 85 228 events passed a mass cut enhancing  $\eta$  mesons,  $m(\pi^-\pi^+\pi^0) < 0.65$  GeV/c<sup>2</sup>, and 74 549 passed a cut to remove events passing through a low-efficiency region in the drift chambers. A final kinematic fit selected 23 492 events for the partial wave analysis (PWA), which were consistent with the  $\eta\pi^0n$ ,  $\eta \rightarrow \pi^+\pi^-\pi^0$  hypothesis at a minimum confidence level of 1%.

A strong  $\eta$  meson signal is observed in this final data sample (Fig. 1a) with a mass of  $539.2 \pm 0.3$  MeV/c<sup>2</sup> and a (resolution-dominated) width of  $23.7 \pm 0.2$  MeV/c<sup>2</sup>. The filled regions in the figure indicate the signal region and the side-band regions used in the analysis. In the  $\eta$  signal region, the signal-to-background ratio is about 6 to 1. The  $\eta\pi^0$  mass spectrum shown in Fig.1b has two clear peaks: the  $a_0^0(980)$  and the  $a_0^0(1320)$ .

The non- $\eta$  background was estimated as a function of  $\eta\pi^0$  mass using the side-band and signal regions. The background fraction varies between 24% and 14% going from lower to higher mass in the region  $0.78 < m(\eta\pi^0) < 1.74$  GeV/c<sup>2</sup>.

The experimental acceptance was determined using a Monte Carlo event sample generated with isotropic angular distributions in the Gottfried-Jackson frame. The detector simulation was based on the E852 detector simulation package SAGEN [12]. The experimental acceptance was incorporated into the PWA by means of Monte Carlo normalization integrals [12]. The acceptance as a function of mass and as a function of  $t'$  is flat.

## PARTIAL WAVE ANALYSIS

The partial-wave analysis (PWA) method described in [12] (see also [21, 22]) was used to study the spin-parity

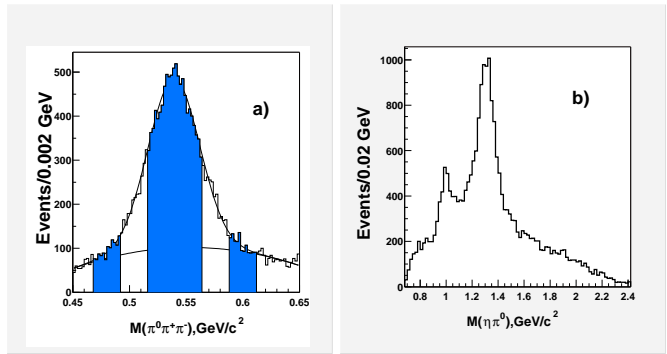


FIG. 1: (a) Fit of the  $\pi^+\pi^-\pi^0$  mass distribution in the  $\eta$  mass region. There are two entries per event: one for each way to assign a  $\pi^0$  to the  $\eta$  decay. The  $\eta$  signal region and the side-band regions are shown shaded. (b) The uncorrected  $\eta\pi^0$  effective-mass distribution for events consistent with the reaction  $\pi^-p \rightarrow \eta\pi^0n$

structure of the  $\eta\pi^0$  system in this data set. The PWA was carried out using the extended maximum likelihood method separately in each mass bin in the mass region between 0.78 and 1.74 GeV in mass bins of 0.04 GeV for  $0 < |t'| < 1.0$  (GeV/c)<sup>2</sup> using the likelihood function

$$\ln \mathcal{L} \propto \sum_i^n \ln I(\Omega_i) - \int d\Omega \eta(\Omega) I(\Omega). \quad (1)$$

Here  $I(\Omega)$  is the predicted angular distribution,  $\eta(\Omega)$  is the angular acceptance, and the sum is over the event sample.

The partial waves are parameterized by a set of five numbers:  $J^{PC}m^\epsilon$ , where  $J$  is the angular momentum,  $P$  and  $C$  are the parity and the C-parity of the  $\eta\pi^0$  system,  $m$  is the absolute value of the angular momentum projection and  $\epsilon$  is the reflectivity. We use a simplified notation where each partial wave is denoted by a letter indicating the  $\eta\pi^0$  system's angular momentum in standard spectroscopic notation, and a subscript which can take the values 0, +, or -, for  $m^\epsilon = 0^-, 1^+, \text{ or } 1^-$  respectively. We assume that the contribution from partial waves with  $m > 1$  is small and can be neglected [12].

The amplitudes used are the unnatural parity-exchange waves (UNPW)  $S_0, P_0, P_-, D_0, D_-$ , and the natural parity-exchange waves (NPW)  $P_+, D_+$ . The NPW waves interfere between themselves as do the UNPW waves but the NPW waves do not interfere with the UNPW waves. The  $P_+$  wave would be an exotic  $J^{PC} = 1^{-+}$  (denoted by  $\pi_1^0$ ) if the wave is resonant.

For each partial wave the complex production amplitudes were determined from an extended maximum likelihood fit [22]. The spin 1/2 nature of the target proton leads to spin-flip and spin-non-flip amplitudes and thus to a production spin-density matrix with maximal rank

two. The PWA fit presented in this paper was carried out with the assumption that a spin-density matrix of rank one was sufficient [12]. An isotropic incoherent background was included. The magnitude of the background was fixed as determined from the side bands. We investigate the quality of the fits by comparing the moments of the decay angular distributions  $H(LM)$ ,  $L \leq 4$  [12, 22], of the data with those predicted by Monte Carlo events generated with the fit amplitudes. We also directly compare the angular distributions for  $\cos(\theta_{GJ})$  and  $\varphi_{TJ}$  between the data and those Monte Carlo events. The quality of the fits is good.

Since natural-parity exchange (NPE) and unnatural-parity exchange (UNPE) amplitudes have different  $|t'|$  dependences, a fit to the  $|t'|$  distribution using a function of the form  $N(t') = n_1|t'|e^{-b_1|t'|} + n_2e^{-b_2|t'|}$  was carried out to determine the relative contributions of the two exchanges. The fitted parameters are  $b_1 = (7.41 \pm 0.08)(\text{GeV}/c)^2$ ,  $b_2 = (2.68 \pm 0.07)(\text{GeV}/c)^2$ , and thus the ratio of UNPE and NPE contributions is equal to  $0.71 \pm 0.03$ . A value about 70% for the ratio of UNPE to NPE is expected in the Regge model at 18 GeV/c.

### AMBIGUOUS SOLUTIONS

There is mathematical ambiguity in the description of a system of two pseudo-scalar mesons [23]. For our set of amplitudes there are eight ambiguous solutions, each of which leads to identical angular distributions. These solutions were found analytically starting from one solution by means of the Barrelet zeros method [22]. The eight solutions in each mass bin are shown in Fig. 2 as a point for every ambiguous solution. (In some cases the solutions are too close together to be visible as separate solutions.) The spread between the various ambiguous solutions dominates the systematic uncertainty in the resonance parameter determination (see below).

### MASS DEPENDENT FITS

To study the resonant structure in the partial waves, we used three different procedures. The first method utilizes a Mass Dependent Fit (MDF) of the average solutions in the NPW sector. The PWA results in each mass bin were averaged between ambiguous solutions [12]. The mass dependence of the  $P_+$  and  $D_+$  intensities as well as their relative phase difference were then fit by relativistic Breit-Wigner (BW) functions (in both the  $P_+$  and  $D_+$  waves) with mass-dependent widths and Blatt-Weisskopf barrier factors. The mass and width of the  $a_0^2$  are well known and were fixed using values[24]:  $M = 1320 \text{ MeV}/c^2$ , and  $\Gamma = 120 \text{ MeV}/c^2$ . The width of  $a_2(1320)$  includes the experimental mass resolution. There are three free parameters in the fit of  $D_+$  intensity,

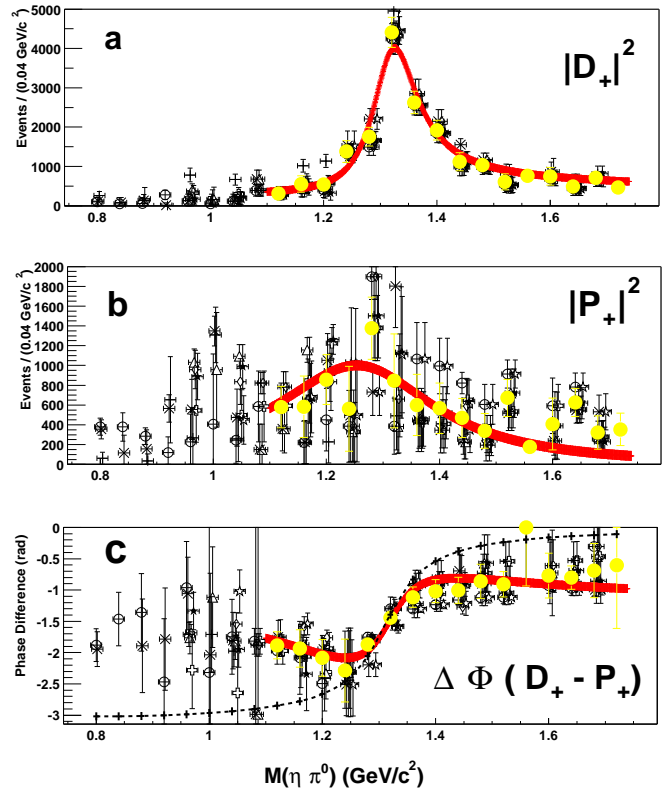


FIG. 2: The Partial Wave Analysis (PWA) and Mass-Dependent Fit (MDF) results. The points shown in each mass bin are the eight ambiguous PWA solutions. a) the  $D_+$  wave intensity; b) the  $P_+$  wave intensity; and c) the relative phase between the  $P_+$  and  $D_+$  waves. The lines show the MDF results. The average PWA solution in each mass bin is plotted using grey points. The dotted line in (c) is the phase difference if the  $P_+$  phase is constant.

$|D_+|^2$ : one for the magnitude and two parameterizing the smooth background for the  $D_+$  wave, as was done in [12].

In the MDF of the  $|P_+|^2$  distribution and the relative phase  $\Delta\Phi(D_+ - P_+)$  there are four free parameters: three from BW function, one for the production phase (assumed constant). The fit was carried out in the mass interval  $1.1 - 1.74 \text{ GeV}/c^2$ . The resonant hypothesis for  $D_+$  and  $P_+$  waves with a mass-independent production phase gives a  $\chi^2/\text{DoF}=1.14$  for 28 degrees of freedom. The non-resonant hypothesis (no phase variation for the  $P_+$  wave) gives  $\chi^2/\text{DoF}=3.02$ . It is clear from Fig. 2c that a single resonant phase for the  $a_2(1320)$  (dotted line) with a constant (non-resonant)  $P_+$  wave is not satisfactory.

The  $P_+$  resonant parameters from the fit with the average solutions and the average error matrix [12] are:  $M = 1265 \pm 20 \text{ MeV}/c^2$  and  $\Gamma = 411 \pm 64 \text{ MeV}/c^2$ .

The second method was similar to the first except that instead of fitting the average solutions, a large number ( $\approx 10^3$ ) of randomly chosen combinations of am-

biguous solutions in each mass bin were used as input to the mass-dependent fit. The obtained distributions of the mass and width of the  $P_+$  resonance were then fitted by a Gaussian. The mean and RMS values of these distributions are:  $M = 1257 \pm 25$  MeV/ $c^2$  and  $\Gamma = 354 \pm 58$  MeV/ $c^2$ . The corresponding curves are shown in Fig. 2.

The third method used was to carry out a Mass-Dependent Partial Wave Analysis (MDPWA) [12]. In this procedure, an extended maximum likelihood function is generalized to include not only the angular distribution, but also the  $\eta\pi^0$  mass distribution for each wave. This analysis is free from the problem of ambiguous solutions but it is necessary to parameterize the mass dependence of every partial wave (including the UNPWs) and all relative phases. We use the same parametrization for the  $D_+$  and  $P_+$  waves as in the first two methods. The mass dependence of the UNPW waves were chosen to be polynomials of second order with constant phases except for the  $S_0$  wave. The  $S_0$  wave was fitted with a BW function using the  $a_0(980)$  resonance parameters. The MDPWA results for the  $P_+$  wave are  $M = 1256 \pm 10$  MeV/ $c^2$  and  $\Gamma = 319 \pm 34$  MeV/ $c^2$ .

In [12] it was shown that a pure  $D_+$  wave can artificially induce a  $P_+$  wave due to the effects of finite acceptance and resolution. This "leakage" leads to a  $P_+$  wave that mimics the  $D_+$  intensity and phase. In our case, the  $P_+$  intensity would therefore have an intensity with the shape of the  $a_2(1320)$  and a  $P_+-D_+$  phase difference which doesn't depend of mass. These features allowed us to include in the MDPWA fit a leakage term with these features. We observed that the leakage contribution to the  $P_+$  wave from the  $D_+$  wave is negligible.

Evidence for a resonance interpretation for the  $P_+$  wave is primarily the behavior of the  $P_+ - D_+$  relative phase (Fig. 2c). Since the  $D_+$  phase variation is well known because of the  $a_2(1320)$  production, it is clear that the  $P_+$  phase cannot be constant (see dotted curve in Fig. 2c) and it is well-described by a BW phase variation.

The ratio of the  $P_+$  and  $D_+$  intensities in the range  $1.24 < M(\eta\pi^0) < 1.34$  GeV is equal to  $|P_+|^2/|D_+|^2 = 0.43 \pm 0.10$ . This ratio is larger than that for the  $\eta\pi^-$  system, as reported in Ref. [12].

The mass of the neutral exotic  $1^{-+}$  state, decaying into  $\eta\pi^0$ , observed here ( $M = 1257 \pm 20 \pm 25$  MeV/ $c^2$ ) is lower than the mass observed in the Crystal Barrel experiment [14] ( $M = 1360 \pm 25$  MeV/ $c^2$ ) by about 100 MeV although the results are consistent within errors. The width measured here ( $\Gamma = 354 \pm 64 \pm 58$  MeV/ $c^2$ ) is also consistent with that from the Crystal Barrel measurement ( $\Gamma = 220 \pm 90$  MeV/ $c^2$ ).

It should also be noted that our result is similar to those obtained from the low- $t'$  fits in Ref. [19]. They obtained  $M = 1301 \pm 14$  MeV/ $c^2$  and  $\Gamma = 190 \pm 32$  MeV/ $c^2$  in one analysis, and  $M = 1386 \pm 32$  MeV/ $c^2$  and  $\Gamma =$

$363 \pm 81$  MeV/ $c^2$  in a fit to the experimental moments. No systematic errors were given for either method.

The lower mass found in our analysis may be a consequence of interference between the resonant state and background in the  $\eta\pi^0$  system, some of which may be from rescattering between the  $\eta$  and the  $\pi^0$ . It is also possible that two or more the resonant  $1^{-+}$  states may be present in the  $\eta\pi^0$  decay channel in the mass interval mass between 1200 and 1400 MeV as might be expected if the exotic state is a four quark state.

## CONCLUSIONS

Mass dependent fits of the  $D_+$  and  $P_+$  amplitudes and their relative phase using three different methods (described above) all lead to the conclusion that the  $P_+$  wave is well-described by a resonance hypothesis and is inconsistent with having a constant phase. The resonance parameters for the observed  $\pi_1^0$  are given by  $M = 1257 \pm 20 \pm 25$  MeV/ $c^2$  and  $\Gamma = 354 \pm 64 \pm 58$  MeV/ $c^2$ . (Here we have chosen to take the resonant parameters and systematic uncertainties from method 2 and the statistical errors from method 1.) This result, together with the previous results from Crystal Barrel [14] and E852 [19] provide strong indications for one or more spin-exotic mesons near 1400 MeV/ $c^2$  decaying to  $\eta\pi^0$ .

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