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Analysis of the $\eta\pi^0$ System with the Decay $\eta \to \pi^+\pi^-\pi^0$

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The exclusive reaction $\pi^- p \to \eta\pi^0 n$ (where $\eta \to \pi^+\pi^-\pi^0$) at 18 GeV/c has been studied in Brookhaven experiment E852. Mass-dependent and mass-independent partial wave analyses have been performed on a sample of 23 492 $\eta\pi^0 n$ events. The analyses yield consistent resonant parameters for the P_+ wave, providing evidence for a neutral exotic meson with $J^{PC} = 1^{-+}$, a mass of $1.270 \pm 0.014^{+0.080}_{-0.070}$ GeV/c^2 and a width of $0.334 \pm 0.042^{+0.116}_{-0.184}$ GeV/c^2 decaying to $\eta\pi^0$.

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Exotic mesons with quantum numbers $J^{PC} = 0^{--}, 1^{-+}, 2^{+-}, \ldots$ do not mix with $q\bar{q}$ mesons and thus offer a natural testing ground for QCD. Exotic mesons have been discussed [1–10] for many years but have only recently been observed experimentally. The underlying structure of the negatively charged exotic state with $J^{PC} = 1^{-+}$ observed in this experiment [11, 12] at 1400 MeV decaying into $\eta\pi^-$ is not yet understood.

An important characteristic of the $\eta\pi^0$ system, unlike the $\eta\pi^-$ system, is that C-parity is a good quantum number. The other distinguishing feature is that the production mechanism for the charge exchange reaction $\pi^- p \to \eta\pi^0 n$ cannot involve the exchange of an isospin $I = 0$ system and thus pomeron exchange is ruled out. These characteristics make the $\eta\pi^0$ system an excellent one to clarify the properties of this exotic state.

The Crystal Barrel experiment [13] confirmed the existence of resonant structure in the $\eta\pi^-$ system using stopped antiprotons in liquid deuterium in the reaction $\bar{p}n \to \pi^-\pi^0\eta$. Later this group analyzed the data on $\bar{p}p$ annihilation at rest into $\pi^0\pi^0\eta$ [14] and presented evidence for an exotic 1^{-+} resonance in the $\eta\pi^0$ system with $M = (1360 \pm 25)$ MeV/c^2 and $\Gamma = (220 \pm 90)$ MeV/c^2.

The $\eta\pi^0$ state has been studied in the GAMS experiment [15] using the reaction $\pi^- p \to \eta\pi^0 n$, $\eta \to 2\gamma$, $\pi^0 \to 2\gamma$ at 32, 38 and 100 GeV/c. Using the method of Sadovsky [16] to resolve the ambiguities in their amplidude analysis, they were able to present evidence for the $\pi_1(1400)$ exotic state in their 38 GeV/c data.

The VES experiment also observed a peak in the P_+ wave of the $\eta\pi^0$ system near 1400 MeV/c^2 [17]. In their most recent publication [18], using theoretical arguments the authors state that the peak can be understood without requiring an exotic meson.

An analysis of E852 data using the reaction $\pi^- p \to \eta\pi^0 p$ (with $\eta \to 2\gamma$) was recently reported [19]. A bump in the P_+ wave of the $\eta\pi^0$ system was observed at $M(\eta\pi^0) = 1272$ MeV/c^2 with a large width ($\Gamma = 660$ MeV/c^2) for all regions of t'. Because of the large width and the uncertainties due to the mathematically ambiguous solutions, the authors chose not to claim evidence for exotic $\pi_1(1400)$ meson production.

In the present analysis we have studied the reaction $\pi^- p \to \eta\pi^0 n$ at 18 GeV/c in E852, using the charged $\eta \to \pi^+\pi^-\pi^0$ decay. The advantage of this mode over the all-neutral final state, where $\eta \to 2\gamma$, is that the production vertex point is defined by charged tracks. This improves the mass resolution as well as the ability to require that the interaction took place in the liquid hydrogen target. We used two independent analyses, one of which is free from the problem of ambiguous solutions. Both analyses give consistent results.

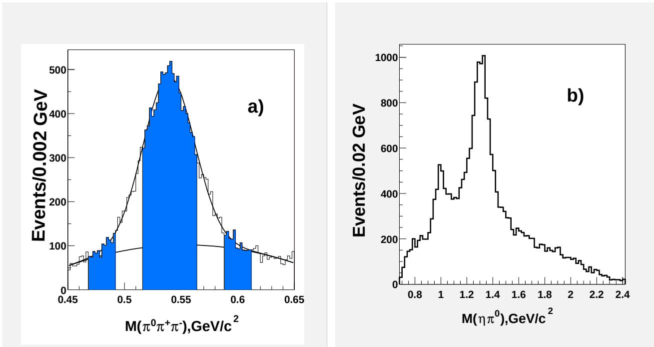

FIG. 1: (a) Fit of the $\pi^+\pi^-\pi^0$ mass distribution in the η mass region. There are two entries per event: one for each way to assign a π^0 to the η decay. The η signal and side-band regions used in the analysis are shown shaded. The signal to background ratio is about 6 to 1. (b) The uncorrected $\eta\pi^0$ effective-mass distribution for events consistent with the reaction $\pi^- p \to \eta\pi^0 n$. The $\eta\pi^0$ mass spectrum has two clear peaks: the $a_0^0(980)$ and the $a_2^0(1320)$.

The data for this analysis was obtained at BNL's Alternating Gradient Synchrotron, where an 18 GeV/c π^- beam interacted in a liquid hydrogen target A total of 750 million triggers were acquired, of which 108 million were of a type designed to enrich the exclusive final state $\pi^- p \to \pi^+\pi^- 4\gamma n$. A total of 6 million events of this type were fully reconstructed. The data were kinematically fit [20] to select events consistent with the $\pi^-\pi^+\pi^0\pi^0 n$ hypothesis (with a confidence level of at least 0.01%) yielding about 4 million events. Of those, 85 228 events passed a mass cut enhancing η mesons, $m(\pi^-\pi^+\pi^0) < 0.65$ GeV/c^2, and 74 549 passed a cut to remove events passing through a low-efficiency region in the drift chambers. A final kinematic fit selected 23 492 events for the partial wave analysis (PWA), which were consistent with the $\eta\pi^0 n, \eta \to \pi^+\pi^-\pi^0$ hypothesis at a minimum confidence level of 1%. A strong η meson signal is observed in this final data sample (Fig. 1a) with a mass of 539.2 ± 0.3 MeV/c^2 and a width of 23.7 ± 0.22 MeV/c^2.

The non-η background was estimated as a function of $\eta\pi^0$ mass using the side-band and signal regions. The background fraction varies between 24% and 14% going from lower to higher mass in the region $0.78 < m(\eta\pi^0) < 1.74$ GeV/c^2. In the mass region 1.10-1.42 GeV, the anisotropy of the angular distributions of the background events is 25% and 15% for $\cos(\theta_{GJ})$ and φ_{TJ} respectively.

The experimental acceptance was determined using a Monte Carlo event sample generated with isotropic angular distributions in the Gottfried-Jackson frame. The detector simulation was based on the E852 detector simulation package SAGEN [12]. The experimental acceptance was incorporated into the PWA by means of Monte Carlo normalization integrals [12]. The acceptance as a function of mass and t' is flat.

The mass-independent partial-wave analysis (PWA) method described in [12] (see also [21, 22]) was used to study the spin-parity structure of the $\eta\pi^0$ system in this data set. The partial waves are parameterized by a set of five numbers: $J^{PC}m^{\epsilon}$, where J is the angular momentum, P and C are the parity and the C-parity of the $\eta\pi^0$ system, m is the absolute value of the angular momentum projection and ϵ is the reflectivity. We use a simplified notation where each partial wave is denoted by a letter indicating the $\eta\pi^0$ system's angular momentum in standard spectroscopic notation, and a subscript which can take the values 0, $+$, or $-$, for $m^{\epsilon} = 0^-$, 1^+, or 1^- respectively. We assume that the contribution from partial waves with $m > 1$ is small and can be neglected [12].

The amplitudes used are the unnatural parity-exchange waves (UNPW) S_0, P_0, P_-, D_0, D_-, and the natural parity-exchange waves (NPW) P_+, D_+. The NPW waves interfere between themselves as do the UNPW waves but the NPW waves do not interfere with the UNPW waves. The P_+ wave would be an exotic $J^{PC} = 1^{-+}$ π_1 if the wave is resonant.

For each partial wave the complex production amplitudes were determined separately for each 0.04 GeV mass bin between 0.78 and 1.74 GeV for $0 < |t'| < 1.0$ (GeV/c)^2 from an extended maximum likelihood fit [22]. The spin 1/2 nature of the target proton leads to spin-flip and spin-nonflip amplitudes and thus to a production spin-density matrix with maximal rank two. The PWA fit presented in this paper was carried out with the assumption that a spin-density matrix of rank one was sufficient [12]. An isotropic incoherent background was included. The magnitude of the background was fixed as determined from the side bands.

There are discrete mathematical ambiguities in the description of a system of two pseudoscalar mesons [23]. For our set of amplitudes there are eight ambiguous solutions, each of which leads to identical angular distributions.

We investigate the quality of the fits by comparing the moments of the decay angular distributions $H(LM), L \leq 4$ [12, 22], of the data with those predicted by Monte Carlo events generated with the fit amplitudes. We also directly compare the angular distributions for $\cos(\theta_{GJ})$ and φ_{TJ} between the data and those Monte Carlo events. The quality of the fits is good.

The spreads between the various ambiguous solutions for the UNPW and P_+ waves are very large. These spreads are however relatively small for the D_+ wave and the relative phase between D_+ and P_+ waves. The intensities associated with these waves for the PWA fit are shown as the points with error bars in Figs. 2, 3, and 4.

To find resonant structure in the partial waves, we carried out a Mass-Dependent Fit (MDF) in the NPW sector. The PWA results in each mass bin were averaged between ambiguous solutions as were the values of the error matrix [12]. The mass dependence of the average values of the P_+ and D_+ intensities (Fig. 2) as well

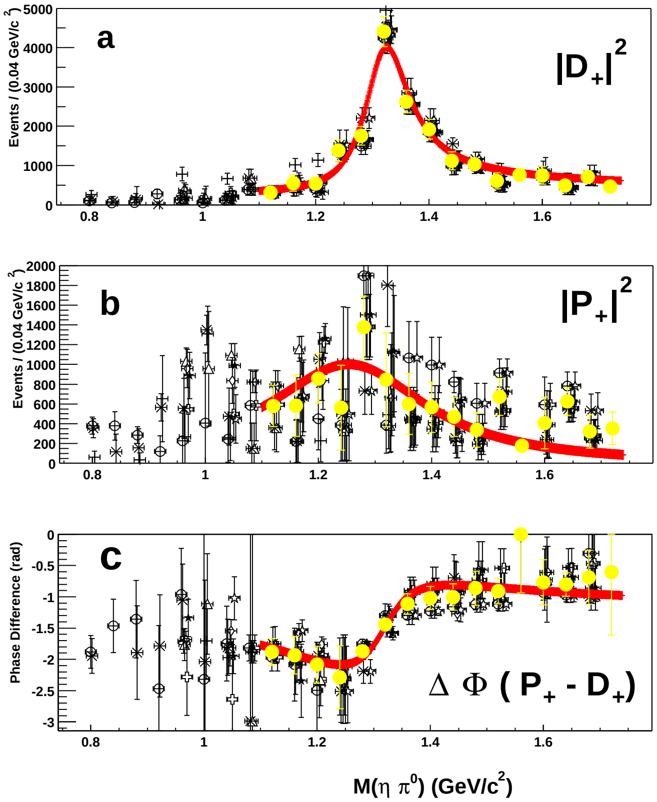

FIG. 2: MDF results (red lines) from fitting the averages of the ambiguous PWA solutions (yellow points with average errors). Fit is in the mass region $1.1 - 1.74$ GeV/c^2. Other points shown are the various ambiguous solutions. a)D_+ wave intensity, b)P_+ wave intensity and c)relative phase of P_+ and D_+ waves.

as their relative phase difference were then fit by relativistic Breit-Wigner (BW) functions (in both the P_+ and D_+ waves) with mass-dependent widths and Blatt-Weisskopf barrier factors. In the MDF there are nine free parameters: six from two BW functions, one for the production phase (assumed constant) and two parameterizing the smooth background for the D_+ wave, as done in [12]. The resonant hypothesis for $D+$ and $P+$ waves with a mass-independent production phase gives a χ^2/DoF=1.22. The non-resonant hypothesis (no phase variation for the $P+$ wave) gives χ^2/DoF=3.02.

Resonant parameters for the fit are given in Table I and the fit is shown as the smooth curves in Fig. 2. The resonant parameters in the D_+ wave are consistent with accepted values for the $a_2(1320)$[24]. (The width of the $a_2(1320)$ includes experimental resolution effects.)

The first error in Table I is statistical, determined using the averaged covariance matrix of the mass-independent PWA; the second is systematic. A large number ($\simeq 10^3$) of randomly chosen combinations of ambiguous solutions in each mass bin were used as inputs to the mass-dependent fits. The spreads in the resonance parameters from these fits give us the systematic error range. Also included in the systematic errors is the choice of the mass region used in the fitting. For example, the change of mass-region to the larger interval $0.78 - 1.74$ GeV/c^2 leads to the parameters of the π_1 (mass $= 1273 \pm 17$ MeV/c^2, width $= 412 \pm 57$ MeV/c^2) which are consistent with the values shown in Table I.

We have carried out a second, independent analysis in an attempt to determine the robustness of our results. This second analysis, a so-called Mass-Dependent Partial Wave Analysis (MDPWA) [12] is free from the problem of ambiguous solutions and thus it is not necessary to take an average of ambiguous solutions or to select between them, as is necessary in the MDF. A potential weakness of the MDPWA is the large number of free parameters needed to parameterize the mass dependence of every wave and all relative phases. In this analysis, the maximum number of free parameters was equal to 22. The MDPWA of the $\eta\pi^0$ system was carried out for $0.78 < m(\eta\pi^0) < 1.74$ GeV/c^2 and $0 < |t'| < 1.0$ (GeV/c)^2. The same set of UNPW and NPW amplitudes used in the PWA are used in the MDPWA. The extended maximum likelihood function is

$$\ln\mathcal{L} \propto \sum_i^n \ln I(\Omega_i, m_i) - \int \mathrm{d}\Omega \mathrm{d}m\ \eta(\Omega, m)\, I(\Omega, m), \quad (1)$$

where m is the $\eta\pi^0$ effective mass, $I(\Omega, m)$ is the predicted angular distribution, $\eta(\Omega, m)$ is the angular acceptance, and the sum is over the event sample. The angular distribution $I(m, \theta, \varphi)$ of the $\eta\pi^0$ system is

$$\begin{aligned} \frac{1}{4\pi}\Big\{ \Big| & S_0(m) + \sqrt{3}P_0(m)d^1_{00}(\theta) + \sqrt{5}D_0(m)d^2_{00}(\theta) \\ & + [\sqrt{6}P_-(m)d^1_{10}(\theta) + \sqrt{10}D_-(m)d^2_{10}(\theta)]\cos\varphi\Big|^2 \\ & + \Big|[\sqrt{6}P_+(m)d^1_{10}(\theta) + \sqrt{10}D_+(m)d^2_{10}(\theta)]\sin\varphi\Big|^2 \\ & + LK(m, \theta, \varphi)\Big\}\ q(m) + BG(m), \end{aligned} \quad (2)$$

where the $d^l_{m0}(\theta)$ are rotation matrices [12], $q(m)$ is the $\eta\pi^0$ break-up momentum, and $BG(m)$ is a smooth, isotropic background term, calculated and fixed using the side band regions shown in Fig. 1a. The mass-dependence of the D_+, P_+, and S_0 amplitudes are taken to be

$$\begin{aligned} P_+(m) &= a_1\Delta(m, m^0_1, \Gamma^0_1)B_1(q)e^{i\ \alpha_1}, & (3) \\ D_+(m) &= a_2\Delta(m, m^0_2, \Gamma^0_2)B_2(q) \times \\ & \times\ [1 + b_1(m - m^0_2) + b_2(m - m^0_2)^2]^{1/2}, & (4) \\ S_0(m) &= a_0\Delta(m, m_0, \Gamma_0), & (5) \end{aligned}$$

where α_1 is the relative production phase between the $P_+(m)$ and $D_+(m)$ waves, a_1, b_1, etc. are fit-parameters, and the relativistic Breit-Wigner amplitude is

$$\Delta(m, m_k, \Gamma_k) = m^0_k\Gamma^0_k/[m^2 - (m^0_k)^2 + im^0_k\Gamma_k(m)]. \quad (6)$$

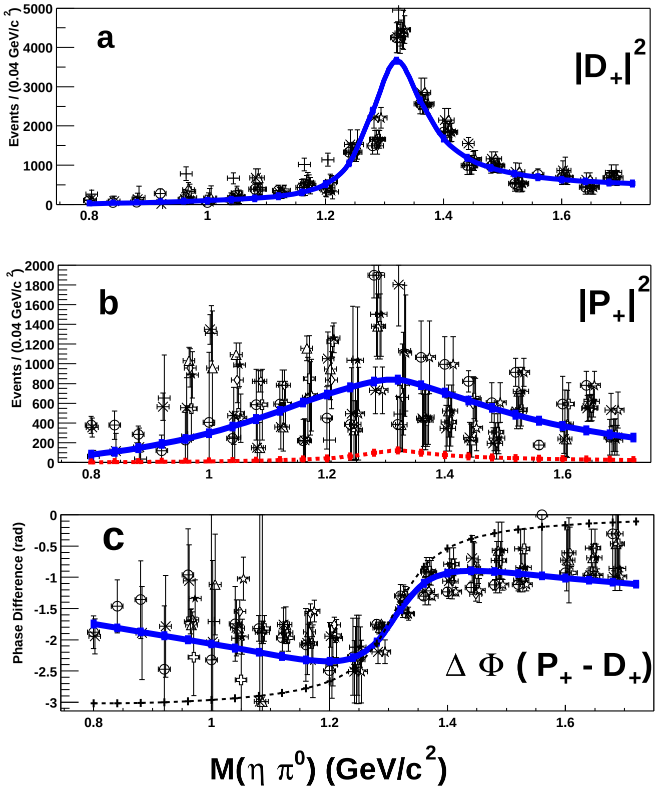

FIG. 3: The results of the MDPWA for the $D+$ and $P+$ waves and the phase difference between them. a) D_+ wave intensity, b) P_+ wave intensity with a leakage contribution (dotted line), and c) the relative phase $(P_+ - D_+)$. The dotted line in (c) is the D_+ phase if there is no P_+ phase variation.

The widths $\Gamma_k(m)$ are well-known functions of mass, which are proportional to the parameter Γ_k^0 and to a Blatt-Weisskopf barrier factor B_i [12].

It has been shown [12] that a pure D+ wave can artificially induce a P_+ signal due to acceptance and resolution effects. Monte Carlo simulation of the E852 resolution shows that this leakage leads to a P_+ intensity with the same mass dependence as the D_+ intensity but with a $(P_+ - D_+)$ phase difference which is independent of mass. In addition, the relative phase between the P_+ wave and the leakage amplitude is close to 90°, and thus can be treated incoherently. These features allow us to introduce a term describing leakage into the P_+ wave from the D_+ wave that has the mass and phase dependence of the $a_2(1320)$ wave, thus

$$LK(m,\theta,\varphi) = |(a_{lk}/a_2)D_+(m)|^2\,[\sqrt{6}d^1_{10}(\theta)\sin\varphi]^2, \quad (7)$$

where the normalization factor a_{lk} describes the intensity of the leakage. The best fit determined $|a_{lk}|/|a_1| = 0.17 \pm 0.02$.

A fit which used resonant shapes (6) for all waves was not satisfactory. Better results were obtained by fitting the small UNPWs with various mass-dependent forms

TABLE I: Results from the MDF analysis

Partial Wave	Mass, MeV/c^2	Width, MeV/c^2
D_+	$1320 \pm 3^{+10}_{-7}$	$96 \pm 3^{+40}_{-15}$
P_+	$1270 \pm 14^{+80}_{-70}$	$334 \pm 42^{+116}_{-184}$

TABLE II: Results from the MDPWA

Partial Wave	Mass, MeV/c^2	Width, MeV/c^2
D_+	$1314 \pm 3^{+13}_{-10}$	$112 \pm 5^{+45}_{-18}$
P_+	$1286 \pm 11^{+40}_{-80}$	$532 \pm 46^{+190}_{-213}$

$W(m)$. The form that worked best is

$$W(m) = a[1 - (m-b)^2/(m-m_{th})^2]e^{i\varphi}, \quad (8)$$

where $m_{th} = m_{\pi^0} + m_\eta$ is the threshold mass. The parameters a, b, and φ are free parameters, determined separately for each wave. Forms such as (8) were tried for the $D_0(m)$ wave, but the best results were obtained when it was allowed to have a resonant shape with the same mass and width as $D_+(m)$, thus

$$D_0(m) \; = \; (a_{D_0}/a_2)D_+(m)e^{i\varphi_{D_0}}. \quad (9)$$

Minima of the likelihood function for all the various fits tried were similar. Table II shows the result of the best fit.

Results of the MDPWA are presented as the smooth curves in Figs. 3 and 4, along with the results (with all ambiguous solutions) of the mass independent PWA for comparison. It must be emphasized that the MDPWA curves are not fit to the mass-independent PWA results. It is clear from Fig. 3c that a single resonant phase for the $a_2(1320)$ (dotted line) with a constant (non-resonant) P_+ wave is not satisfactory.

The systematic errors for the D_+ and P_+ resonance parameters given in Table II are determined from the spread in the resonance parameters between the fits with different assumptions about the UNPW mass dependences. The parameters from the MDF analysis given in Table I and those from the MDPWA analysis given in Table II are consistent.

Evidence in favor of a resonance interpretation for the P_+ wave is the behavior of the $P_+ - D_+$ phase difference (Fig.3c) and the difference in the quality of the fit for a resonant (χ^2/DoF = 1.22) and non-resonant (χ^2/DoF = 3.02) P_+ wave. The ratio of the P_+ and D_+ intensities in the range $1.24 < M(\eta\pi^0) < 1.34$ GeV is equal to $|P_+|^2/|D_+|^2 = 0.43 \pm 0.10$. This ratio is larger than that for the $\eta\pi^-$ system, as reported in Ref. [12]. Our study of the leakage contribution to the P_+ wave from the D_+ wave shows that it is very small.

The mass of the neutral exotic 1^{-+} state, decaying into $\eta\pi^0$, observed here (1270 MeV) is lower than the mass observed in the Crystal Barrel experiment (1360 MeV) by about 100 MeV although the results are consistent

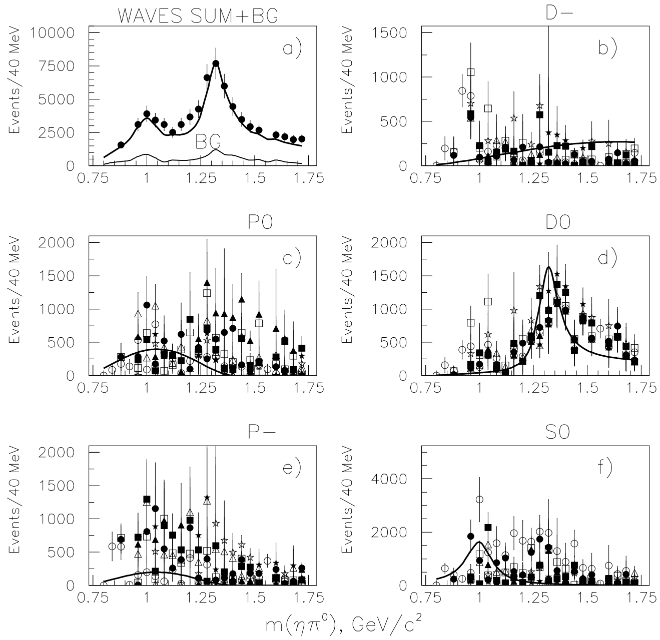

FIG. 4: The results of the MDPWA for the unnatural parity waves. a) A sum of all waves and the background intensity, BG, b) D_- wave intensity, c) P_0 wave intensity, d) D_0 wave intensity, which was fit with the same BW resonant parameters as the D_+ wave, e) P_- wave intensity, and f) S_0 wave intensity. The P_0, P_-, and D_- waves were fit by a 2nd order polynomial with a constant phase.

within errors. The lower mass found here may be a consequence of interference between the resonant state and background in the $\eta\pi^0$ system, some of which may be from rescattering between the η and the π^0. Another interpretation is that this exotic state may belong to a four quark decuplet of SU(3) with a particular mixing angle.

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