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Analysis of the $\eta\pi^0$ System with the Decay $\eta \to \pi^+\pi^-\pi^0$

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The exclusive reaction $\pi^-p \to \eta\pi^0 n$ (where $\eta \to \pi^+\pi^-\pi^0$) at 18 GeV/c has been studied in Brookhaven experiment E852. Mass-dependent and mass-independent partial wave analyses have been performed on a sample of 23,492 $\eta\pi^0 n$ events. The analyses yield consistent resonant parameters for the P_+ wave,providing evidence for a neutral exotic meson with $J^{PC} = 1^{-+}$, a mass of $1.270 \pm 0.014^{+0.080}_{-0.070}$ GeV/c^2 and a width of $0.334 \pm 0.042^{+0.116}_{-0.184}$ GeV/c^2 decaying to $\eta\pi^0$.

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INTRODUCTION

Exotic mesons with quantum numbers $J^{PC} = 0^{--}, 1^{-+}, 2^{+-}, \ldots$ do not mix with quark-antiquark mesons and thus offer a natural testing ground for QCD. Exotic mesons have been discussed [1–10] for many years but have only recently been observed experimentally. The underlying structure of the negatively charged exotic state with $J^{PC} = 1^{-+}$ observed in the experiment [11, 12] at 1400 MeV decaying into $\eta\pi^-$ is not yet understood.

Study of the resonant structure of the neutral $\eta\pi^0$ system near 1400 MeV can be very important in attempting to understand this underlying structure. An important characteristic of the $\eta\pi^0$ system, unlike the $\eta\pi^-$ system, is that C-parity is a good quantum number. The other distinguishing feature is that the production mechanism for the charge exchange reaction $\pi^-p \to \eta\pi^0 n$ cannot involve the exchange of an isospin $I = 0$ system and thus pomeron exchange is ruled out. These characteristics make the $\eta\pi^0$ system an excellent one to clarify the properties of this exotic state.

The Crystal Barrel experiment [13] confirmed the existence of resonant structure in the $\eta\pi^-$ system using stopped antiprotons in liquid deuterium in the reaction $\bar{p}n \to \pi^-\pi^0\eta$. Later this group analyzed the data on $\bar{p}p$ annihilation at rest into $\pi^0\pi^0\eta$ [14] and presented evidence for an exotic 1^{-+} resonance in the $\eta\pi^0$ system with $M = (1360 \pm 25)$ MeV/c^2 and $\Gamma = (220 \pm 90)$ MeV/c^2.

The $\eta\pi^0$ state has been studied in the GAMS experiment [15] using the reaction $\pi^-p \to \eta\pi^0 n$, $\eta \to 2\gamma$, $\pi^0 \to 2\gamma$ at 32, 38 and 100 GeV/c. They showed that the intensity of the P_+ wave has a wide bump at $M = 1300$ MeV/c^2. This structure was difficult to characterize because of the presence of ambiguities in the amplitude analysis. The statistics of the 38 GeV/c data was sufficient so that the method of Sadovsky [16] could be used to resolve the ambiguity, and they thus were able to present evidence for the $\pi_1(1400)$ exotic state.

The VES experiment also observed a peak in the P_+ wave of the $\eta\pi^0$ system near 1400 MeV/c^2. See a review of their results in [17]. In their most recent publication [18], using theoretical arguments the authors state that the peak can be understood without requiring an exotic meson.

An analysis of E852 data using the reaction $\pi^- p \to \eta\pi^0 p$ with decay $\eta \to 2\gamma$ (instead of $\eta \to \pi^+\pi^-\pi^0$ considered in this article) was recently reported [19]. A bump in the P_+ wave of the $\eta\pi^0$ system was observed at $M(\eta\pi^0) = 1272$ MeV/c^2 with a large width ($\Gamma = 660$ MeV/c^2) in the whole t' region. Because of the large width and the indefinities of mathematically ambiguouos solutions the authors chose not to claim evidence for exotic $\pi_1(1400)$ meson production.

In the present analysis we have studied the reaction $\pi^- p \to \eta\pi^0 n$ at 18 GeV/c in E852, using the charged $\eta \to \pi^+\pi^-\pi^0$ decay. The advantage of this mode over the all-neutral final state is that the production vertex point is defined by charged tracks. This improves the mass resolution as well as the ability to require that the interaction took place in the liquid hydrogen target. We use also two independent methods, one of which is free a problem of ambiguous solutions. Both analysises give the consistent results.

EXPERIMENTAL SETUP AND DATA SELECTION

The data for this analysis was obtained at the Alternating Gradient Synchrotron (BNL USA). Using an 18 GeV/c π^- beam interacting in a liquid hydrogen target, a total of 750 million triggers were acquired of which 108 million were of a type designed to enrich the exclusive final state $\pi^- p \to \pi^+\pi^- 4\gamma n$. A total of 6 million events of this type were fully reconstructed. The data were kinematically fitted [20] to select events consistent with the $\pi^-\pi^+\pi^0\pi^0 n$ hypothesis yielding some 4 million events. After a mass cut enhancing η mesons, $m(\pi^-\pi^+\pi^0) < 0.65$ GeV/c^2, we have 85228 events. Following a cut to remove events passing through a low-efficiency region in the drift chambers, we ended up with a sample of 74,549 events of the type $\pi^-\pi^+\pi^0\pi^0 n$. Then the data were kinematically fitted to select 31,679 events consistent with the $\eta\pi^0 n$ hypothesis. Requiring a minimum acceptable confidence level of 1% for this hypothesis, a total of 23,492 $\eta\pi^0 n$ events remained for the partial wave analysis (PWA).

After the cuts described above, a strong η meson signal is observed (see Fig. 1a) with a mass of 539.2 ± 0.3 MeV/c^2 and a width of 23.7 ± 0.22 MeV/c^2. The filled regions in the figure indicate the side-band regions and the η signal region used in the analysis. The ratio of η signal to background is about 6 to 1. The $\eta\pi^0$ mass spectrum shown in Fig.1b has two clear peaks: the $a_0^0(980)$ and the $a_2^0(1320)$.

The ratio of background (i.e. non-η events) to data events, was estimated using the side-band and signal regions (see Fig. 1a) as a function of $\eta\pi^0$ mass. The ratio varies between 24% and 14% from lower to higher mass in the region $0.78 < m(\eta\pi^0) < 1.74$ GeV/c^2. The unisotropy of angular distribution in the background (region of side bands) is 25% and 15% for $\cos(\theta_{GJ})$ and φ_{TJ} distributions in mass region 1.10-1.42 GeV.

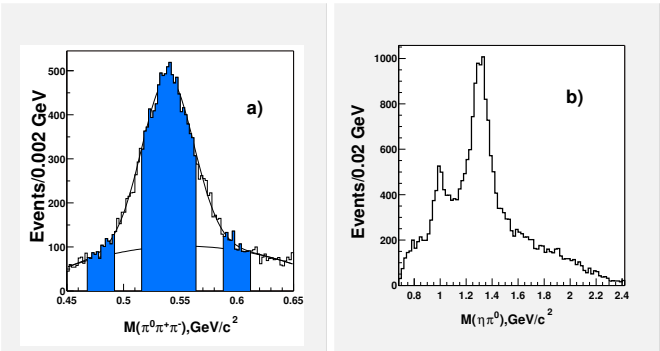

FIG. 1: (a) Fit of $\pi^0\pi^+\pi^-$ mass distribution (two entries per event) in the η mass region. The η signal and the selection of side bands (hatched zones). (b) The uncorrected $\eta\pi^0$ effective mass distribution for events consistent with the reaction $\pi^- p \to \eta\pi^0 n$

The experimental acceptance was determined using a Monte Carlo event sample. The Monte Carlo events were generated with isotropic angular distributions in the Gottfried-Jackson frame. The detector simulation was based on the E852 detector simulation package SAGEN [12]. The experimental acceptance was incorporated into the PWA by means of Monte Carlo normalization integrals [12]. The acceptance as a function of mass and t' is flat.

TABLE I: Fitted BW Resonance Parameters by the help of MDF.

Partial Wave	Mass, MeV/c^2	Width, MeV/c^2
D_+	$1320 \pm 3^{+10}_{-7}$	$96 \pm 3^{+40}_{-15}$
P_+	$1270 \pm 14^{+80}_{-70}$	$334 \pm 42^{+116}_{-184}$

PARTIAL WAVE ANALYSIS IN EACH MASS BIN AND MASS DEPENDENY FIT

A mass-independent partial-wave analysis (PWA) [12, 21, 22] of the data was used to study the spin-parity structure of the $\eta\pi^0$ system. PWA shown in this paper was carried out in the region mass 0.78 to 1.74 with mass bin 0.04 GeV and in $0 < |t'| < 1.0(GeV/c)^2$ region, using the extended maximum likelihood function in each separate mass bin:

$$\ln\mathcal{L} \propto \sum_i^n \ln I(\Omega_i) - \int d\Omega \eta(\Omega)\, I(\Omega), \tag{1}$$

where $I(\Omega_i)$ is a predicted angular distribution and $\eta(\Omega)$ is the angular acceptance.

The partial waves are parameterized by a set of five numbers: $J^{PC}m^\epsilon$, where J is the angular momentum, P and C is a parity and a charged parity of the $\eta\pi^0$ system, m is the absolute value of the angular momentum projection and ϵ is the reflectivity. We will use simplified notation where each partial wave is denoted by a letter, indicating the $\eta\pi^0$ system's angular momentum in standard spectroscopic notation, and a subscript, which can take the values 0, +, or −, for $m^\epsilon = 0^-$, 1^+, or 1^- respectively. We assume that the contribution from partial waves with $m > 1$ is small and can be neglected [12]. The following set of partial waves was used: S_0, P_-, P_0, P_+, D_-, D_0, D_+. For each partial wave the complex production amplitudes are determined from an extended maximum likelihood fit [22]. The spin-flip and spin-nonflip contributions to the baryon vertex lead to a production spin-density matrix with maximal rank two.. The PWA fit presented in this paper was carried out with a spin-density matrix of rank one [12]. An incoherent background was included. It is equal to the data multiplied by the ratio of events in the side bands to η signal. The background was isotropic and fixed.

There are discrete mathematical ambiguities in the description of a system of two pseudoscalar mesons [23]. For our set of amplitudes there are eight ambiguous solutions.

We investigate a quality of the fits in a comparison of the moments $H(LM), L \leq 4$, (see [12], [22]) between the data and the predicted Monte Carlo events. The same is for the angular distributions on $\cos(\theta_{GJ})$ and φ_{TJ}. The quality of the fits is good. This comparison doesn't depend on the ambiguous solutions.

The spreads of the ambiguous solutions are very large for UNPW waves, but are relatively small for the D_+ wave and the relative phase between D_+ and P_+ waves. So we used a method of average solutions [12]. PWA results in each mass bin were averaged between ambiguous solutions and also we used average error matrix. Then the P_+ and D_+ intensities as well as their relative phase difference were fitted (see Fig.(2)). We call this fit as Mass Dependent Fit (MDF).

The data points were fitted by relativistic Breit-Wigner (BW) functions (in both the P_+ and D_+ waves) with mass-dependent widths and Blatt-Weisskopf barrier factors. In MDF we have nine free parameters: 6 from two BW functions, one of production phase and two of smooth background of D_+ wave (see parametrization in [12]). The resonant hypothesis for $D+$ and $P+$ waves with a mass-independent production phase [12] in Table I gives $\chi^2/DoF = 1.22$. The non-resonant hypothesis (no phase for the $P+$ wave) gives $\chi^2/DoF = 3.02$.

The resulting resonance parameters are given in Table I for a non-exotic resonance $a_2(1320)$ in the D_+ wave and for a exotic resonance π_1 the in P_+. The resonant parameters of $a_2(1320)$ meson correspond to the standard parameters [24]. A width of $a_2(1320)$ reflects experimental resolution.

The first error in Table I is statistical, determined using the covariance matrix of the mass-independent PWA; the second is systematic. A large number ($\simeq 10^3$) chosen randomly combinations of ambiguous solutions in each mass bin were used as inputs to the mass-dependent fits. The spreads in the resonance parameters from these fits give us the systematic error range. For example the change of mass region to more large interval $0.78 - 1.74\ GeV/c^2$ leads to the parameters of π_1: mass $= 1273 \pm 17 MeV/c^2$, width $= 412 \pm 57 MeV/c^2$, which is not strong differ from values of Table I. Our study of leakage contribution to P_+ wave from D_+ wave shows that it is very small in MDF.

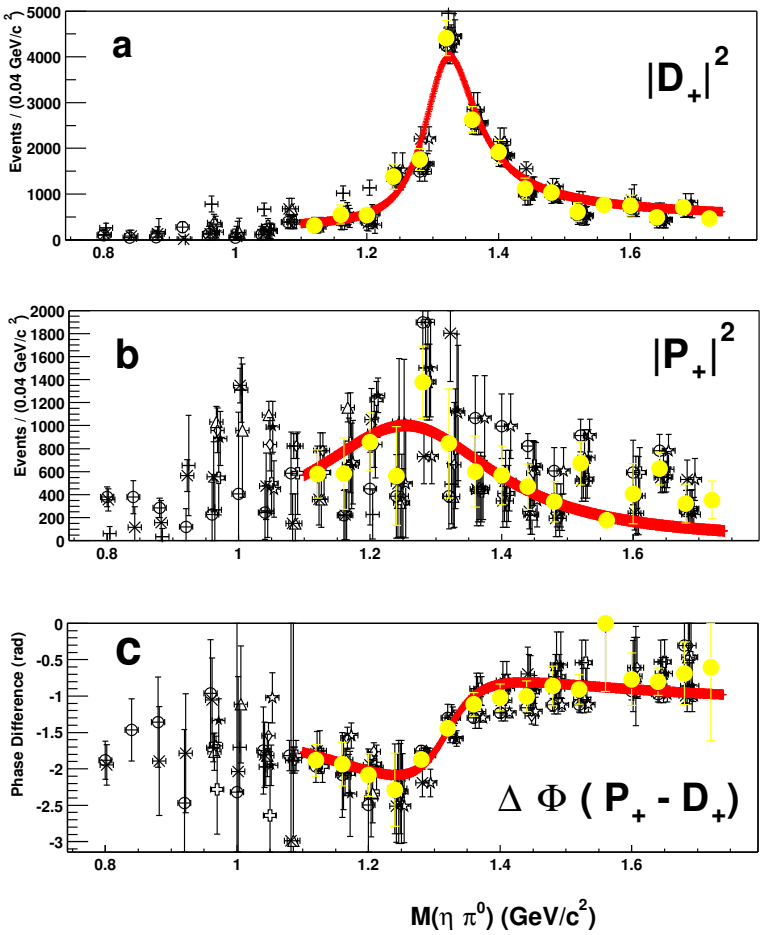

FIG. 2: MDF results of the average ambiguous solutions (yellow points with average errors). Fit is in the mass region $1.1 - 1.74\ GeV/c^2$ (red lines). Other points are ambiguous solutions. a)D_+ wave intensity, b)P_+ wave intensity and c)relative phase of P_+ and D_+ waves.

MASS DEPENDED PARTIAL WAVE ANALYSIS

It is useful to check up the results by other analysis, which is free from a problem of the ambiguous solutions. We used so called Mass Dependent Partial Wave Analysis (MDPWA) [12]. The results of such analysis does not depend on ambiguous solutions and it is not necessary to take an average of ambiguous solutions or to select between them. It is a main advantage of MDPWA. But a weakness of MDPWA is many free parameters needed to parametrize mass dependence of every wave and all relative phases. Maximum number of free parameters was equal to 22. The MDPWA of the $\eta\pi^0$ system was carried out for $0.78 < m(\eta\pi^0) < 1.74$ GeV/c^2 and $0 < |t'| < 1.0$ (GeV/c)^2.

In MDPWA the angular distributions are fitted simultaneously in each $\eta\pi^0$ mass bin (2). The bins are tied together with a mass-dependent function for each partial wave. The same set of amplitude as in PWA was used: S_0, P_0, P_-, D_0, D_- (Unnaturally Parity Waves (UNPW)) and P_+, D_+ (Naturally Parity Waves (NPW)).

In MDPWA the extended maximum likelihood function is generalized to include not only the angular distribution, but also the $\eta\pi^0$ mass distribution for each wave.

$$\ln\mathcal{L} \propto \sum_i^n \ln I(\Omega_i, m_i) - \int d\Omega dm\ \eta(\Omega, m)\, I(\Omega, m). \tag{2}$$

The angular distribution of $\eta\pi^0$ system is:

$$
\begin{aligned}
I(m,\theta,\varphi) &= \frac{1}{4\pi}\{\Big|S_0(m)+\sqrt{3}P_0(m)d^1_{00}(\theta)+\sqrt{5}D_0(m)d^2_{00}(\theta) \\
&+[\sqrt{6}P_-(m)d^1_{10}(\theta)+\sqrt{10}D_-(m)d^2_{10}(\theta)]\cos\varphi\Big|^2 \\
&+\Big|[\sqrt{6}P_+(m)d^1_{10}(\theta)+\sqrt{10}D_+(m)d^2_{10}(\theta)]\sin\varphi\Big|^2 \\
&+LK(m,\theta,\varphi)\}\ q(m) \\
&+BG(m). \qquad (3)
\end{aligned}
$$

Here $LK(m,\theta,\varphi)$ is a leakage of P_+ wave from D_+ wave. It was shown [12] that a pure D+ wave can artificially induce a P_+ wave due to acceptance and resolution effects. This "leakage" leads to a P_+ intensity with the same mass dependence as the D_+ intensity and with a $(P_+ - D_+)$ phase difference which is independent of mass. Our study of leakage by Monte Carlo simulation of E852 resolution has shown that the relative phase between P_+ wave and the leakage is close to 90°, so we included the leakage incoherently. These features allow us to introduce a term describing leakage $LK(m,\theta,\varphi)$ which has the mass dependence of $a_2(1320)$ intensity.

$q(m)$ is the break-up momentum. $BG(m)$ is the smooth and isotropic background, which is calculated with the help of the side bands under the η meson signal and fixed in our fits.

Here we used the next mass dependence (the amplitudes of D_+ and P_+ waves is the same as in [12]):

$$
\begin{aligned}
P_+^{(res)}(m) &= a_1\Delta(m,m_1^0,\Gamma_1^0)B_1(q)e^{i\ \alpha_1}; && (4)\\
D_+(m) &= a_2\Delta(m,m_2^0,\Gamma_2^0)B_2(q)[1+b_1(m-m_2^0)+b_2(m-m_2^0)^2]^{1/2}; && (5)\\
S_0(m) &= a_0\Delta(m,m_0,\Gamma_0); && (6)\\
LK(m,\theta,\varphi) &= \mid P_{lk}(m)\mid^2\ [\sqrt{6}d^1_{10}(\theta)\sin\varphi]^2. && (7)
\end{aligned}
$$

$P_{lk}(m)$ is proportional to the D_+-wave mass dependence and has its own normalization factor a_{lk}.

$$
P_{lk}(m) = a_{lk}\cdot\frac{D_+(m)}{a_2}. \qquad (8)
$$

Breit-Wigner amplitude $\Delta(m,m_k,\Gamma_k)$ is

$$
\Delta(m,m_k,\Gamma_k) = \frac{m_k^0\cdot\Gamma_k^0}{(m^2-(m_k^0)^2)+i(m_k^0\Gamma_k(m))} = e^{i\phi_k(m)}\ \Big|\Delta(m,m_k^0,\Gamma_k^0)\Big|, \qquad (9)
$$

where $\phi_k(m)$ is a BW phase of wave amplitude. The widths $\Gamma_k(m)$ are well known functions of mass, which are proportional to parameter Γ_k^0 and a Blatt-Weisskopf barrier factor (see [12]). α_1 is the relative production phase of interfering waves $P_+(m)$ and $D_+(m)$.

For small UNPWs we used different shape of smooth mass dependence $W(m)$. The shapes of polynomial of second order and polynomial-exponential mass dependence are equal to

$$
\begin{aligned}
W_1(m) &= a(1-(m-b)^2/(m-m_{th})^2)e^{i\varphi} && (10)\\
W_2(m) &= a(m-m_{th})(1+b(m-m_{th}))e^{i\varphi}; && (11)\\
W_3(m) &= a(m-m_{th})^2e^{-b(m-m_{th})}e^{i\varphi} && (12)
\end{aligned}
$$

Here m is the $\eta\pi^0$-mass, $m_{th}=m_{\pi^0}+m_\eta$ is the threshold mass. The wave intensity must go to zero at the threshold mass. Parameters a,b,φ were different for each wave and free.

We made many fits with different shapes of UNPWs mass dependences. For all of them there are the same forms of NPWs $P_+(m)$ (4), $D_+(m)$ (5) and $S_0(m)$ (6). One fit was with the resonant shape (9) of all waves. This fit was not satysfactory. In the different fits $D_0(m)$ wave has a parametrization as smooth mass dependence (10)-(12) or as resonant shape with the same mass and width as $D_+(m)$:

$$
D_0(m) = a_{D_0}\frac{D_+(m)}{a_2}e^{i\varphi_{D_0}} \qquad (13)
$$

Minimums of likelihood function of all fits were close. We demonstrate below the result of best fit with the larger value of $|\ln\mathcal{L}|$ in extremum between different fits. This fit corresponds to the polynomial parametrization

TABLE II: Fitted BW Resonance Parameters by the help of MDPWA.

Partial Wave	Mass, MeV/c^2	Width, MeV/c^2
D_+	$1314 \pm 3^{+13}_{-10}$	$112 \pm 5^{+45}_{-18}$
P_+	$1286 \pm 11^{+40}_{-80}$	$532 \pm 46^{+190}_{-213}$

$W_1(m)$ of P_0, P_-, D_- waves and $D_0(m)$ wave as resonant shape (13). The fits include the whole region of mass $0.78 < m(\eta\pi^0) < 1.74$ GeV/c^2.

MDPWA results (lines) are presented in Fig. 3 and Fig. 4. The lines are not fitted to the points. The points of ambiguous solutions in each mass bin are shown only for visual comparison with the lines, which are calculated with parameters of MDPWA from the fitting of likelihood function (2).

The D_+ and P_+ resonance parameters obtained are given in Table II. The spreads in the resonance parameters between the fits with different assumptions about UNPW mass dependencies give us the estimate of the systematic errors of MDPWA in Table II.

The parameters from the MDF procedure in Table I and from the MDPWA in Table II are consistent in the error limits. A contribution of P_+ wave leakage from D_+ wave is $|a_{lk}|/|a_1| = 0.17 \pm 0.02$. It is clear from Fig. 3c that a single resonant phase of $a_2(1320)$ (dotted line) is not satisfied data relative phase of D_+ and P_+ waves.

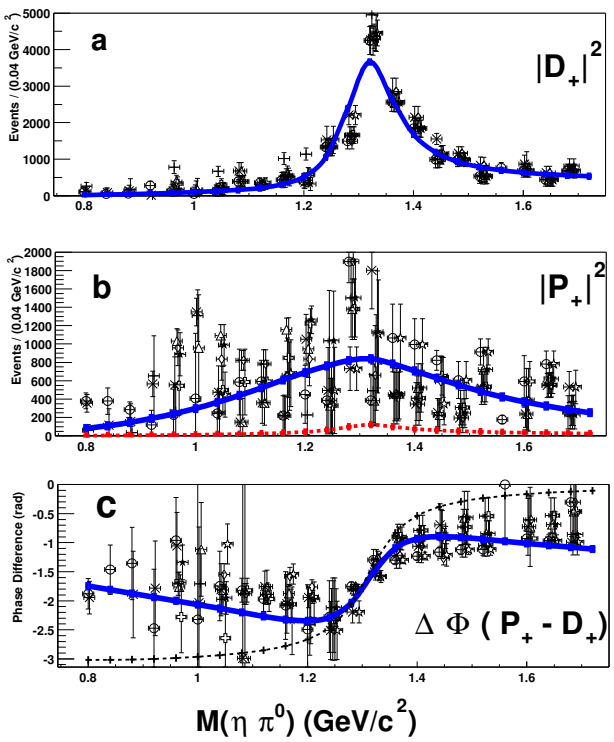

FIG. 3: The results of MDPWA for NPWs $D+$ and $P+$ waves and phase difference between them. a) D_+ wave intensity, b) P_+ wave intensity and a leakage contribution (dotted line), c) the relative phase ($P_+ - D_+$) and the resonant D_+ phase (dotted line)

CONCLUSION

A partial wave analysis of data (23,492 events) collected in experiment E852 from reaction $\pi^- p \to \eta\pi^0 n$ (where $\eta \to \pi^+\pi^-\pi^0$) at 18 GeV/c was performed. Two methods (MDF and MDPWA) of $\eta\pi^0$ mass dependence study give the consistent results of exotic $\pi_1(1400)$ meson with resonant parameters closed to the results published early in the error limit. We emphasise that it is not necessary for selecting the physical solution amoung the mathematically ambiguouos solutions in our MDPWA. The bright distingushing feature of phase mass dependence (Fig.3c) and the difference in $\chi^2/DoF = 1.22$ and 3.02 between the resonant and non-resonant nature of P_+ wave is a strong

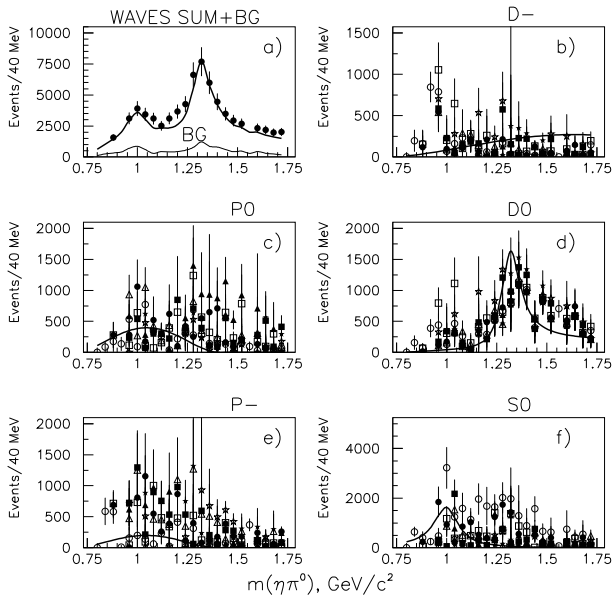

FIG. 4: The results of MDPWA for UNPWs. a) A sum of waves and background BG, b)D_- wave intensity, c)P_0 wave intensity, d)D_0 wave intensity, which was fitted with fixed BW resonant parameters as for D_+ wave, e)P_- wave intensity, f)S_0 wave intensity. The waves P_0, P_-, D_- were fitted as polynomial background with constant phase.

evidence in a favour of $\pi_1(1400)$ exotic meson production. The ratio of the P_+ and D_+ intensities in the range $1.24 < M(\eta\pi^0) < 1.34$ GeV is equal to $|P_+|^2/|D_+|^2 = 0.43 \pm 0.10$. This ratio is larger than that for the $\eta\pi^-$ system [12]. We prefer to show the parameters of the neutral exotic 1^{-+} state in Table I as a the final results for the mass interval $1.10-1.74\,GeV/c^2$because at mass below $1.10\,GeV/c^2$ there are more indefinites, connecting with contribution of ω decay $\omega \to \pi^-\pi^+\pi^0$.

The mass of the neutral exotic 1^{-+} state, decaying into $\eta\pi^0$, observed here (1270 MeV) is lower than the mass observed in the Crystal Barrel experiment (1360 MeV) by about 100 MeV. Within errors the results are consistent however . If in the future our result with lower mass is confirmed in the diffraction production process with larger statistics, then the reason may be as a consequence of interference between the resonant state and background in $\eta\pi^0$ system. A source of the background in the $\eta\pi^0$ system may be rescattering between the η and the π^0. Another interpretation is that this exotic state may belong to a four quark decuplet of SU(3) with a particular mixing angle.

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