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Analysis of the $\eta\pi^0$ System with the Decay $\eta \to \pi^+\pi^-\pi^0$

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The exclusive reaction $\pi^-p \to \eta\pi^0 n$, $\eta \to \pi^+\pi^-\pi^0$ at 18 GeV/c has been studied in Brookhaven experiment E852. Mass-dependent and mass-independent partial wave analyses have been performed on a sample of 23,492 $\eta\pi^0 n$ events. The analyses yield consistent resonant parameters for the P_+ wave,providing evidence for a neutral exotic meson with $J^{PC} = 1^{-+}$, a mass of $1.270 \pm 0.014^{+0.080}_{-0.070}$ GeV/c^2 and a width of $0.334 \pm 0.042^{+0.116}_{-0.184}$ GeV/c^2 decaying to $\eta\pi^0$.

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INTRODUCTION

Exotic mesons with quantum numbers $J^{PC} = 0^{--}, 1^{-+}, 2^{+-}, \ldots$ do not mix with quark-antiquark mesons and thus offer a natural testing ground for QCD. Exotic mesons have been discussed [1–10] for many years but have only recently been observed experimentally. The underlying structure of the negatively charged exotic state with $J^{PC} = 1^{-+}$ observed in the experiment [11, 12] at 1400 MeV decaying into $\eta\pi^-$ is not yet understood [13].

Study of the resonant structure of the neutral $\eta\pi^0$ system near 1400 MeV can be very important in attempting to understand this underlying structure. An important characteristic of the $\eta\pi^0$ system, unlike the $\eta\pi^-$ system, is that C-parity is a good quantum number. The other distinguishing feature is that the production mechanism for the charge exchange reaction $\pi^-p \to \eta\pi^0 n$ cannot involve the exchange of an isospin $I = 0$ system and thus pomeron exchange is ruled out. These characteristics make the $\eta\pi^0$ system an excellent one to clarify the properties of this exotic state.

The Crystal Barrel experiment [14] confirmed the existence of resonant structure in the $\eta\pi^-$ system using stopped antiprotons in liquid deuterium in the reaction $\bar{p}n \to \pi^-\pi^0\eta$. Later this group analyzed the data on $\bar{p}p$ annihilation at rest into $\pi^0\pi^0\eta$ [15] and presented evidence for an exotic 1^{-+} resonance in the $\eta\pi^0$ system with $M = (1360 \pm 25)$ MeV/c^2 and $\Gamma = (220 \pm 90)$ MeV/c^2.

The $\eta\pi^0$ state has been studied in the GAMS experiment [16] using the reaction $\pi^-p \to \eta\pi^0 n$, $\eta \to 2\gamma$, $\pi^0 \to 2\gamma$ at 32, 38 and 100 GeV/c. They showed that the intensity of the P_+ wave has a wide bump at $M = 1300$ MeV/c^2. This structure was difficult to characterize because of the presence of ambiguities in the amplitude analysis. The statistics of the 38 GeV/c data was sufficient so that the method of Sadofsky [17] could be used to resolve the ambiguity, and they thus were able to present evidence for the $\pi_1(1400)$ exotic state.

The VES experiment also observed a peak in the P_+ wave of the $\eta\pi^0$ system near 1400 MeV/c^2. See a review of their results in [18]. In their most recent publication [19], using theoretical arguments the authors state that the peak can be understood without requiring an exotic meson.

An analysis of E852 data using the reaction $\pi^-p \to$

$\eta\pi^0 p$ with the all-neutral decay $\eta \to 2\gamma$ (instead of $\eta \to \pi^+\pi^-\pi^0$ as reported in this work) was recently reported [20]. A bump in the P_+ wave of the $\eta\pi^0$ system was observed at $M(\eta\pi^0) = 1272$ MeV/c^2 with a large width ($\Gamma = 660$ MeV/c^2). Because of the large width and the strong t' dependence of the P_+ resonant parameters, the authors chose not to claim evidence for exotic $\pi_1(1400)$ meson production.

In the present analysis we have studied the reaction $\pi^- p \to \eta\pi^0 n$ at 18 GeV/c in E852, using the charged $\eta \to \pi^+\pi^-\pi^0$ decay . The advantage of this mode over the all-neutral final state is that the production vertex point is defined by charged tracks. This improves the mass resolution as well as the ability to require that the interaction took place in the liquid hydrogen target.

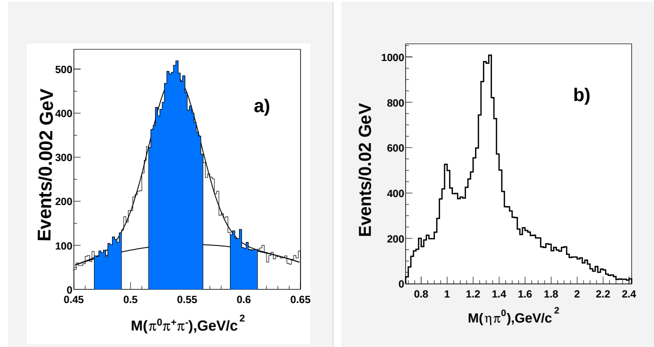

FIG. 1: (a) Fit of $\pi^0\pi^+\pi^-$ mass distribution (two entries per event) in the η mass region. The η signal and the selection of side bands (hatched zones) . (b) The uncorrected $\eta\pi^0$ effective mass distribution for events consistent with the reaction $\pi^- p \to \eta\pi^0 n$

EXPERIMENTAL SETUP AND DATA SELECTION

The data for this analysis was obtained at the Alternating Gradient Synchrotron (BNL USA). Using an 18 GeV/c π^- beam interacting in a liquid hydrogen target, a total of 750 million triggers were acquired of which 108 million were of a type designed to enrich the exclusive final state $\pi^- p \to \pi^+\pi^- 4\gamma n$. A total of 6 million events of this type were fully reconstructed. The data were kinematically fitted [21] to select events consistent with the $\pi^-\pi^+\pi^0\pi^0 n$ hypothesis yielding some 4 million events. After a mass cut enhancing η mesons, $m(\pi^-\pi^+\pi^0) < 0.65$ GeV/c^2, we have 85228 events. Following a cut to remove events passing through a low-efficiency region in the drift chambers, we ended up with a sample of 74,549 events of the type $\pi^-\pi^+\pi^0\pi^0 n$. Then the data were kinematically fitted to select 31,679 events consistent with the $\eta\pi^0 n$ hypothesis. Requiring a minimum acceptable confidence level of 1% for this hypothesis, a total of 23,492 $\eta\pi^0 n$ events remained for the partial wave analysis (PWA).

After the cuts described above, a strong η meson signal is observed (see Fig. 1a) with a mass of 539.2 ± 0.3 MeV/c^2 and a width of 23.7 ± 0.22 MeV/c^2. The filled regions in the figure indicate the side-band regions and the η signal region used in the analysis. The ratio of η signal to background is about 6 to 1. The $\eta\pi^0$ mass spectrum shown in Fig.1b has two clear peaks: the $a_0^0(980)$ and the $a_2^0(1320)$.

The ratio of background (i.e. non-η events) to data events, was estimated using the side-band and signal regions (see Fig. 1a) as a function of $\eta\pi^0$ mass. The ratio varies between 15% and 25% in the region $0.78 < m(\eta\pi^0) < 1.74$ GeV/c^2.

The experimental acceptance was determined using a Monte Carlo event sample. The Monte Carlo events were generated with isotropic angular distributions in the Gottfried-Jackson frame. The detector simulation was based on the E852 detector simulation package SAGEN [12]. The experimental acceptance was incorporated into the PWA by means of Monte Carlo normalization integrals [12]. The acceptance as a function of mass and t' is rather flat .

The acceptance-corrected distribution of the four-momentum-transfer $|t'|$ is shown in Fig. 2. Since natural-parity exchange (NPE) and unnatural-parity exchange (UNPE) amplitudes have different $|t'|$ dependence, a fit to a function of the form $N(t') = n_1|t'|e^{-b_1|t'|} + n_2 e^{-b_2|t'|}$ was carried out to determine the relative contributions of the two. The fitted parameters are $b_1 = (7.41 \pm 0.08)(\text{GeV/c})^2$, $b_2 = (2.68 \pm 0.07)(\text{GeV/c})^2$, $n_2/n_1 = 0.71 \pm 0.03$. Here n_1 and n_2 is proportional to the number of events produced by NPE and UNPE, accordingly. A value about 70% for the ratio of UNPE to NPE is expected in the Regge model at 18 GeV/c.

The amplitude analysis discussed below was carried out for the data in the range $0.0 < |t'| < 1.0$ GeV$^2/c^2$. In addition, PWAs were carried out for two intervals of t'. Here $t'_{eq} = 0.225$ GeV$^2/c^2$ was chosen as the boundary between the low and high regions of t' in order to divide the sample into approximately equal numbers of events.

PARTIAL WAVE ANALYSIS AND MASS DEPENDENT FIT

A mass-independent partial-wave analysis (PWA) [12, 22, 23] of the data was used to study the spin-parity structure of the $\eta\pi^0$ system. The partial waves are parameterized by a set of five numbers: $J^{PC}m^\epsilon$, where J is the angular momentum, P the parity and C the C parity of the $\eta\pi^0$ system; m is the absolute value of the angular momentum projection; and ϵ is the reflectivity. We will use simplified notation in which each partial wave is denoted by a letter, indicating the $\eta\pi^0$ system's angu-

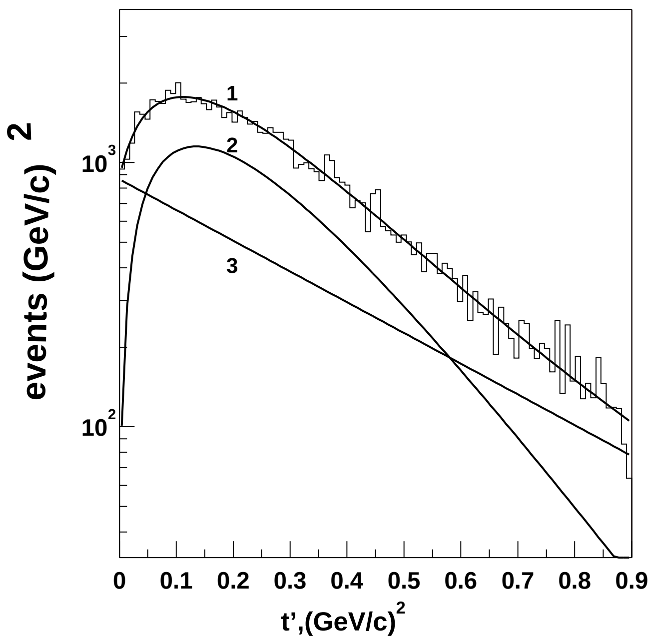

FIG. 2: The acceptance corrected t' - distribution and the results of fit. 1) Sum = NPE + UNPE, 2) NPE contribution , 3) UNPE contribution.

lar momentum in standard spectroscopic notation, and a subscript, which can take the values 0, +, or −, for $m^{\epsilon} = 0^-$, 1^+, or 1^- respectively. We assume that the contribution from partial waves with $m > 1$ is small and can be neglected [12].

Mass-independent PWA fits shown in this paper are carried out in 0.04 mass bins from 1.1 to 1.74 GeV/c^2 and 0.10 GeV/c^2 mass bins from 0.7 to 1.74 GeV/c^2 and all use the S_0, P_-, P_0, P_+, D_-, D_0, D_+ partial waves. For each partial wave the complex production amplitudes are determined from an extended maximum likelihood fit [23]. The spin-flip and spin-nonflip contributions to the baryon vertex lead to a production spin-density matrix with maximal rank two. A rank two mass-independent PWA in a system of two pseudoscalars cannot be performed because of the presence of a continuous mathematical ambiguity. The PWA fits presented in this paper are carried out with a spin-density matrix of rank one. An incoherent background was included. It is equal to the data multiplied by the ratio of events in the side bands to η signal. The background was isotropic and fixed. The experimental resolution has not been unfolded.

Results of the PWA are shown as the points with statistical errors in Fig. 3 and Fig. 5 for the 40 MeV mass bin size and in Fig. 4 for two t' intervals using 100 MeV bins. There are discrete mathematical ambiguities in the description of a system of two pseudoscalar mesons [24]. For our set of amplitudes there are eight ambiguous solutions. They are shown as points with recalculated statistical errors in Fig. 3 and in Fig. 5. Data in Fig. 4 correspond to the average ambiguous solutions.

The spreads of ambiguous solutions are very large for UNPW (Fig. 3), but it is not the same for D_+ wave and a relative phase between D_+ and P_+ waves (Fig. 5).

We investigate a quality of the fits in a comparison of the moments $H(LM), L \leq 4$, (see [12], [23]) between the data and the predicted Monte Carlo events. The same is for the angular distributions on $\cos(\theta_{GJ})$ and φ_{TJ}. The quality of the fits is good. This comparison doesnt depend on the ambiguous solutions.

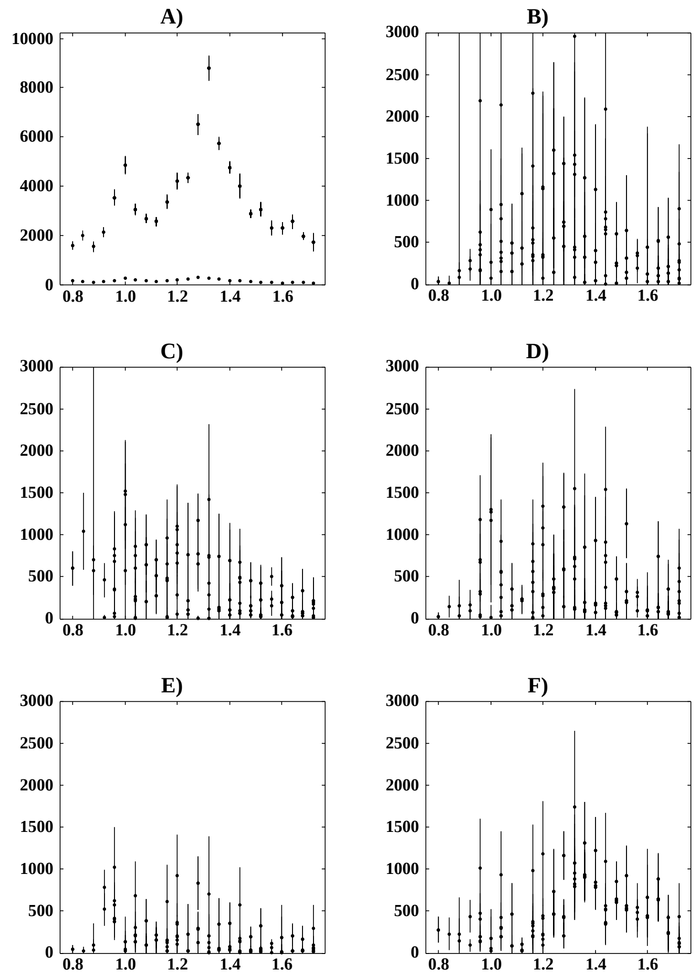

FIG. 3: PWA results for the intensities A) sum and background and UNPW ambiguous solutions of B) S_0, C) P_-, D) P_0, E) D_-, F) D_0 waves as a function of mass $\eta\pi^0$ system, GeV/c^2 with 40 MeVc^2 mass bin.

To study the nature of the observed peaks a mass-dependent fit (MDF) of PWA results have been carried out [12]. These results were obtained with averaged ambiguous solutions and average error matrix. The P_+ and D_+ intensities from the PWA as well as their phase difference were fitted. The data points were fitted by relativistic Breit-Wigner functions (in both the P_+ and D_+ waves) with mass-dependent widths and Blatt-Weisskopf barrier factors [12]. The resonant hypothesis for $D+$ and

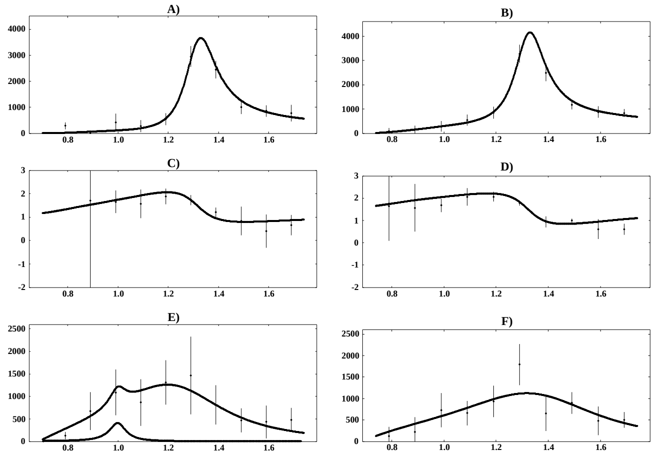

FIG. 4: The results of mass dependent fit of the intensities of D_+ and P_+ waves and phase between them at $0 < t' < 0.225(GeV/c)^2$ (left panel) and $0.225 < t' < 1.0(GeV/c)^2$ (right panel) with 100 MeV mass bin. A) and B) are the intensities of D_+ waves, C) and D) $(P_- - D_+)$ relative phase, E) and F) are the intensities of P_- waves. Small low bump in fig. E) is a leakage of S_0 wave.

TABLE I: Fitted Resonance Parameters in 40 MeV mass bins with (PWA and MDF) and without the average ambiguous solutions (MDPWA).

Partial Wave	Mass, MeV/c^2	Width, MeV/c^2
$D_+, (PWA\ and\ MDF)$	$1320 \pm 3^{+10}_{-7}$	$96 \pm 3^{+40}_{-15}$
$P_+, (PWA\ and\ MDF)$	$1270 \pm 14^{+80}_{-70}$	$334 \pm 42^{+116}_{-184}$
$D_+, (MDPWA)$	1326 ± 3	112 ± 5
$P_+, (MDPWA)$	1283 ± 7	382 ± 24

$P+$ waves with a mass-independent production phase [12] in Table I gives $\chi^2/DoF = 1.22$. The non-resonant hypothesis for the $P+$ wave gives $\chi^2/DoF = 3.02$.

The resulting resonance parameters are given in Table I for the P_+ exotic resonance and a non-exotic resonance in the D_+ wave. The first error in Table I is statistical, determined using the covariance matrix of the mass-independent PWA; the second is systematic. A large number ($\simeq 10^3$) chosen randomly combinations of ambiguous solutions in each mass bin were used as inputs to the mass-dependent fits. The spreads in the resonance parameters from these fits give us the most large systematic errors.

It was shown [12] that a pure D+ wave can artificially induce a P_+ wave due to acceptance and resolution effects. This "leakage" leads to a P_+ intensity with the same mass dependence as the D_+ intensity and with a $(P_+ - D_+)$ phase difference which is independent of mass. These features allow us to introduce in the MDF a term describing leakage (with the "leakage intensity" (I_{leak})) which is equal to the known mass dependence

TABLE II: Results of the mass dependent fit in different t'-ranges in 100 MeV mass bins with the average ambiguous solutions.

Parameters	all region	$t' < t'_{eq}$	$t' > t'_{eq}$
	PWA+MDF	100 MeV mass bin	
$M(a_2), MeV/c^2$	1327 ± 5	1320 ± 12	1326 ± 6
$\Gamma(a_2), MeV/c^2$	146 ± 8	151 ± 30	142 ± 8
$M(\pi_1), MeV/c^2$	1286 ± 40	1246 ± 51	1364 ± 50
$\Gamma(\pi_1), MeV/c^2$	585 ± 92	507 ± 117	612 ± 97
I_{leak_D+}	< 0.001	< 0.001	< 0.001
I_{leak_S0}	333 ± 732	407 ± 592	< 0.001
χ^2/DoF	1.3	0.83	1.1
	MDPWA	40 MeV mass bin	
$M(\pi_1), MeV/c^2$	1283 ± 7	1289 ± 10	1269 ± 14
$\Gamma(\pi_1), MeV/c^2$	382 ± 24	381 ± 28	493 ± 44

$a_2(1320)$ intensity multiplied on a factor I_{leak}. The intensity of leakage from the D_+ wave was found to be: $I_{leak}(D_+) < 0.001$ and $I_{leak}(D_+)/I(D_+) \sim 10^{-7}$.

In order to determine whether the fit parameters depend on t', we select two separate four-momentum-transfer intervals above and below $t'_{eq} = 0.225(GeV/c)^2$. The fitting was then carried out in 100 MeV mass bins to increase statistic in mass bin interval. The results of the PWA fit using 23,492 events in the whole t'-range and mass range $0.74 < M(\eta\pi^0) < 1.74\ GeV/c^2$ were compared with the results of the PWA fits of 12,062 events in $t' < t'_{eq}$ range and 11,430 events in $t' > t'_{eq}$ range (see Table II). Leakage from the D_+ and S_0 waves were included in the fits. The systematic errors for the π_1^0 parameters shown in Table II include uncertainty due to the presence of leakage from the D+ wave. This leakage is much less than that from the S_0 wave in the region of the $a_0(980)$. The S_0 wave leakage is large $(I_{leak}(S_0)/I(S_0) \sim 0.5)$. The similar study was made in MDPWA (see the results in Table II).

The resonance masses in different t'-ranges are consistent (Table II). The widths in analysis with 100 MeV mass bin are larger than that obtained in the 40 MeV bin but are approximately consistent within errors inside the same method of analysis.

MASS DEPENDENT PWA

It is usefull to check up the results by other analysis, which is free from the ambiguous solutions. We used so called Mass Dependent PWA (MDPWA) [12]. The results of such analysis does not lead to ambiguous solutions so it is not necessary to take an average of ambiguous solutions or to select between them.

For this MDPWA all mass bins are fitted simultaneously rather than carrying out a separate PWA for each

$\eta\pi^0$ mass bin. The bins are tied together with a mass-dependent function for each partial wave. The likelihood function then depends not only on the angular distribution, but also on the $\eta\pi^0$ mass distribution for each wave.

Unfortunally MDPWA has many free parameters. The fit with all free parameters is unstable. So at start we take the parameters of NPW from (PWA+MDF) procedure. Then we fit data and find UNPW parameters. The forms of their mass dependences are taken to describe approximately background of P_0, P_-, D_- waves and the resonant behaviour of S_0 and D_0 waves. Then we fix UNPW parameters and fit again data with free parameters of NP waves D_+ and P_+.

A sources of systematic errors in MDPWA are the various forms of UNP waves mass dependences. Since they are not known exactly we don't calculate systematic errors in MDPWA.

The results of the MDPWA are shown in Fig. 5. The resonant parameters of NPW are in Table I and in Table II. The parameters from the (PWA+MDF) procedure and from the MDPWA are consistent.

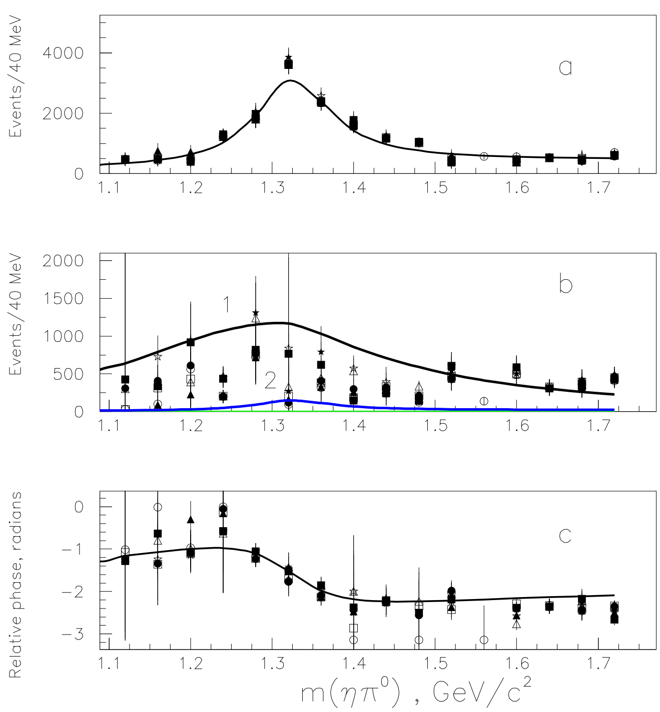

FIG. 5: The results of MDPWA for $|D+|^2$ (a) and $|P+|^2$ (b) waves and phase difference $(P_+ - D_+)$ (c). Line 1 is a sum $|P+|^2$ and the leakage from D_+ wave, line 2 is a leakage .

CONCLUSION

The mass of the neutral exotic 1^{-+} state, decaying into $\eta\pi^0$, observed here (1270 MeV) differs from that observed in the Crystal Barrel experiment (1360 MeV) by about 100 MeV and the widths differ as well (330 MeV and 220 MeV respectively). Within errors the results are consistent however. If in the future our result with lower mass is confirmed in the diffraction production process with larger statistics, then the reason may be as a consequence of interference between the resonant state and background in $\eta\pi^0$ system. A source of the background in the $\eta\pi^0$ system may be rescattering between the η and the π^0. Another interpretation is that this exotic state may belong to a four quark decuplet of SU(3) with a particular mixing angle.

In conclusion, we have studied the $\eta\pi^0$ system produced in the reaction $\pi^- p \rightarrow n\eta\pi^0$ at 18 GeV/cWe find that an exotic meson, the $\pi_1^0(1400)$ is produced, decaying to $\eta\pi^0$. The resonant parameters are consistent in two separate t' intervals as well as in two method of analysis. The ratio of the P_+ and D_+ intensities in the range $1.24 < M(\eta\pi^0) < 1.34$ GeV is equal to $|P_+|^2/|D_+|^2 = 0.35 \pm 0.10$. This ratio is larger than that for the $\eta\pi^-$ system [12].

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[1] R.L. Jaffe and K. Johnson, Phys. Lett. **60B**, 201 (1976); R.L. Jaffe, Phys. Rev. D**15**, 267 (1977).
[2] T. Barnes and F.E. Close, Phys. Lett. B **116**, 365 (1982).
[3] T. Barnes et al., Nucl. Phys. B **224**, 241 (1983).
[4] N. Isgur and J. Paton, Phys. Rev. D **31**, 2910 (1985).
[5] I. Balitsky *et al.*, Z. Phys. C **33**, 265 (1986); J.I. Latorre,

P. Pascual, and S. Narison, Z. Phys, C **34**, 347 (1987); J. Govaerts et al., Nucl. Phys. B **284**, 674 (1987).
[6] T. Barnes *et al.*, Phys. Rev. D **52**, 5242 (1995).
[7] F.E. Close and P.R. Page, Nucl. Phys. B **443**, 233 (1995).
[8] Y. Uehara et al., Nucl. Phys. A **606**, 357 (1996).
[9] P. Lacock, C. Michael, P. Boyle and P. Rowland, Phys. Rev. D **54**, 6997 (1996); C. Bernard et al., Nucl. Phys. B (Proc. Suppl.) **53**, 228 (1997).
[10] T. Barnes, Acta Phys. Polon. **B31**, 2545 (2000).
[11] D.R. Thompson *et al.*, Phys. Rev. Lett. **79**, 1630 (1997).
[12] S.U. Chung *et al.*, Phys. Rev. D **60**, 092001 (1999).
[13] S.U. Chung and E.Klempt , Phys. Lett. B **563**, 83 (2003).
[14] A. Abele *et al.*, Phys. Lett. B **423**, 175 (1998).
[15] A. Abele *et al.*, Phys. Lett. B **446**, 349 (1999).
[16] A.Lednev, Proceedings of 7-th Int. Conf. on Hadron Spectroscopy, Upton, **NY**, 253 (1977)
[17] S.A. Sadovsky, Nucl. Phys. A **655**, 131c (1999).
[18] D. Alde *et al.* Phys. Atom. Nucl., **62**, 421 (1999).
[19] D.V. Amelin *et al.*, Phys.Atom.Nucl. **68**, 359 (2005).
[20] A.R. Dzierba *et al.*, Phys. Rev. D **67**, 094015 (2003).
[21] O.I. Dahl et al., "SQUAW kinematic fitting program", Univ. of California, Note P-126, unpublished (1968).
[22] J. P. Cummings and D. P. Weygand, "The New BNL PWA Programs", BNL-64637, unpublished (1997).
[23] S.U. Chung, Phys. Rev. D **56**, 7299 (1997).
[24] S.A. Sadovsky, Soviet Physics Doklady **36**, 537 (1991).