Study of properties of hot matter created in heavy ion collisions

Egor Nedorezov (JINR, MSU), Alexey Aparin (JINR).

SINP MSU, Strong Interactions Laboratory seminar

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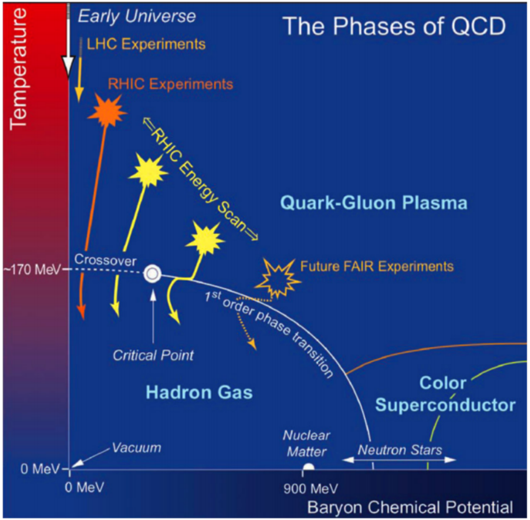
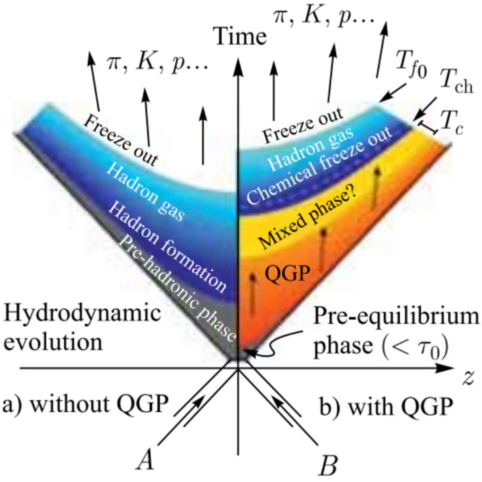
Evolution of quark-gluon plasma

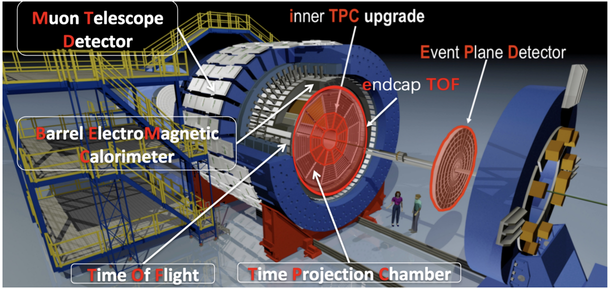




The STAR experiment

TPC (Time Projection Chamber)

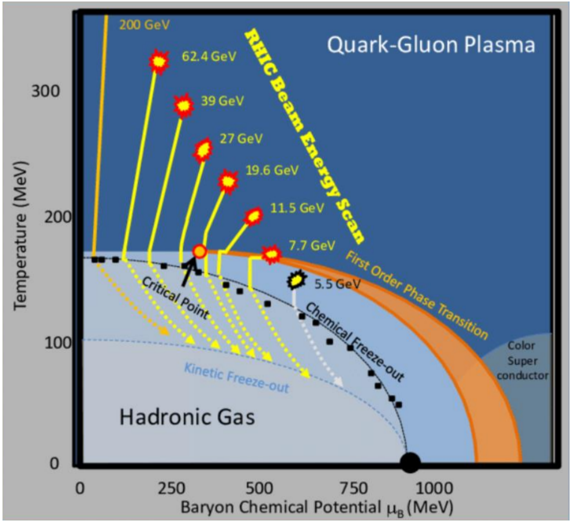
- used for tracking and identification
- length 4.2 m, diameter 4 m (1 m),
- azimuthal angle 2π
- pseudorapidity range $|\eta| < 1$
- in a magnetic field 0.5 Tesla



Beam Energy Scan program on RHIC

Used data:

- RHIC BES-I, 2010-2011
- $Au + Au \sqrt{S_{NN}} = 7.7 - 39$ GeV.
- Phys.Rev.C 96 (2017) 044904,
  Phys.Rev.C 92 (2015) 014904.



General relations

- The general expression for transverse momentum distribution of particles of relativistic ideal gas:

$$\frac{\mathrm{d}^2 N}{p_T \,\mathrm{d}\, p_T \,\mathrm{d}\, y} = \frac{V}{(2\pi)^3} \int\limits_0^{2\pi} \mathrm{d}\,\varphi E_{\vec{p}} \sum_{\sigma} \langle n_{\vec{p}\sigma} \rangle,$$

where $E_{\vec{p}} = m_T \cosh y, \; m_T = \sqrt{p_T^2 + m^2}, \; \langle n_{\vec{p}\sigma} \rangle$ - occupation numbers.

Boltzmann-Gibbs statistics

- Boltzmann-Gibbs statistics:

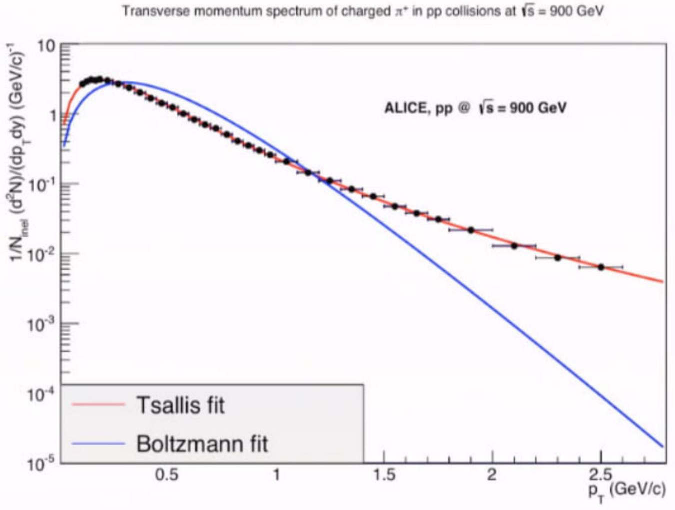
$$\langle n_{\vec{p}\sigma} \rangle = \frac{1}{\exp\left(\frac{E_{\vec{p}}-\mu}{T}\right) + \eta},$$

where $\eta = -1(\text{Bose–Einstein}),\ 0(\text{Maxwell–Boltzmann}),\ 1(\text{Fermi–Dirac})$.

- For $\eta = 0(\text{Maxwell–Boltzmann})$ transverse momentum distribution can be integrated by y analytically:

$$\frac{\mathrm{d}\,N}{p_T\,\mathrm{d}\,p_T} = \frac{V}{2\pi^2} m_T\,\mathrm{K}_1\left(\frac{m_T}{T}\right)$$

Tsallis vs Boltzmann



Presentation by J Cleymans @ CERN Heavy Ion Forum 2014

Existing Tsallis distributions

- Levy function:

  (G. Wilk and Z. W lodarczyk, Phys. Rev. Lett. 84, 2770 (2000))

$$\frac{\mathrm{d}^2 N}{p_T\, \mathrm{d}p_T\, \mathrm{d}y} = \frac{\mathrm{d}N}{\mathrm{d}y} \frac{(n-1)(n-2)}{nC(nC + m_0(n-2))} \left(1 + \frac{m_T - m_0}{mC}\right)^{-n}$$

- Tsallis-like distribution:

  (Alberico W M, Lavagno, Eur. Phys. J. A 40 313 (2009))

$$\frac{\mathrm{d}^2 N}{p_T\, \mathrm{d}p_T\, \mathrm{d}y} = A \left(1 + (q-1)\frac{m_T \cosh y - \mu}{T}\right)^{\frac{1}{1-q}}$$

- Phenomenological Tsallis distribution:

  (Cleymans J, Worku D, J.Phys.G:Nucl.Part.Phys. 39, 025006 (2012))

$$\frac{\mathrm{d}^2 N}{p_T\, \mathrm{d}p_T\, \mathrm{d}y} = \frac{gV}{(2\pi)^2} m_T \cosh y \left(1 + (q-1)\frac{m_T \cosh y - \mu}{T}\right)^{\frac{q}{1-q}}$$

So which model should be used?

Tsallis-3 statistics (J.Phys.G:Nucl.Part.Phys.50 2023 125002)

- Tsallis entropy:

$$S = \sum_i \frac{p_i^q - p_i}{1-q}, \ \sum_i p_i = 1,$$

где p_i - probability ith microscopic state of the system, $q \in [0, \infty]$.

- In the Gibbs limit $q \to 1$ the entropy recovers the Boltzmann-Gibbs entropy:

$$S = \sum_i p_i \ln p_i$$

- In Grand Canonical Ensemble thermodynamic potential Ω takes the form:

$$\Omega = \langle H \rangle - TS - \mu \langle N \rangle, \text{ where}$$

$$\langle H \rangle = \frac{1}{\theta} \sum_i p_i^q E_i, \ \langle N \rangle = \frac{1}{\theta} \sum_i p_i^q N_i, \ \theta = \sum_i p_i^q.$$

Tsallis-3 statistics

Let's consider a relativistic ideal gas in the Grand Canonical Ensemble. From the principle of thermodynamic equilibrium (the principle of maximum entropy), normalization expressions for the parameters $\Lambda = -\theta TS + \langle H\rangle - \mu\langle N\rangle$ и $\theta = \sum_i p_i^q$ can be derived:

$$\begin{cases} 1 = \sum\limits_{n=0}^{n_0} \dfrac{\omega^n}{n!\,\Gamma\left(\frac{1}{q-1}\right)} \int\limits_0^\infty t^{\frac{2-q}{q-1}-n} e^{-t+\beta'(\Lambda+\mu n)} (\mathrm{K}_2(\beta' m))^n \,\mathrm{d}t\,, \\ \theta = \sum\limits_{n=0}^{n_0} \dfrac{\omega^n}{n!\,\Gamma\left(\frac{q}{q-1}\right)} \int\limits_0^\infty t^{\frac{1}{q-1}-n} e^{-t+\beta'(\Lambda+\mu n)} (\mathrm{K}_2(\beta' m))^n \,\mathrm{d}t\,, \end{cases} .$$

where $\omega = \dfrac{gV}{2\pi^2}\dfrac{m^2T\theta^2}{q-1}$, $\beta' = \dfrac{-t(1-q)}{T\theta^2}$,

n_0 - the number of terms to be taken into account, counting from zero.

Tsallis-3 statistics

- The expression for transverse momentum distribution of particles of relativistic ideal gas in the grand canonical ensemble in rapidity range $y \in [y_{\text{min}}, y_{\text{max}}]$ takes the form:

$$\left.\frac{\mathrm{d}^2 N}{p_T\, \mathrm{d}p_T\, \mathrm{d}y}\right|_{y_{\text{min}}}^{y_{\text{max}}} = \frac{gV}{(2\pi)^2} m_T \int\limits_{y_{\text{min}}}^{y_{\text{max}}} \mathrm{d}y \cosh y \times$$

$$\times \frac{1}{\theta} \sum_{n=0}^{n_0} \frac{\omega^n}{n!\, \Gamma\left(\frac{q}{q-1}\right)} \int\limits_0^\infty t^{\frac{1}{q-1}-n} e^{-t+\beta'(\Lambda - m_T \cosh y + \mu(n+1))} (\mathrm{K}_2(\beta' m))^n \, \mathrm{d}t$$

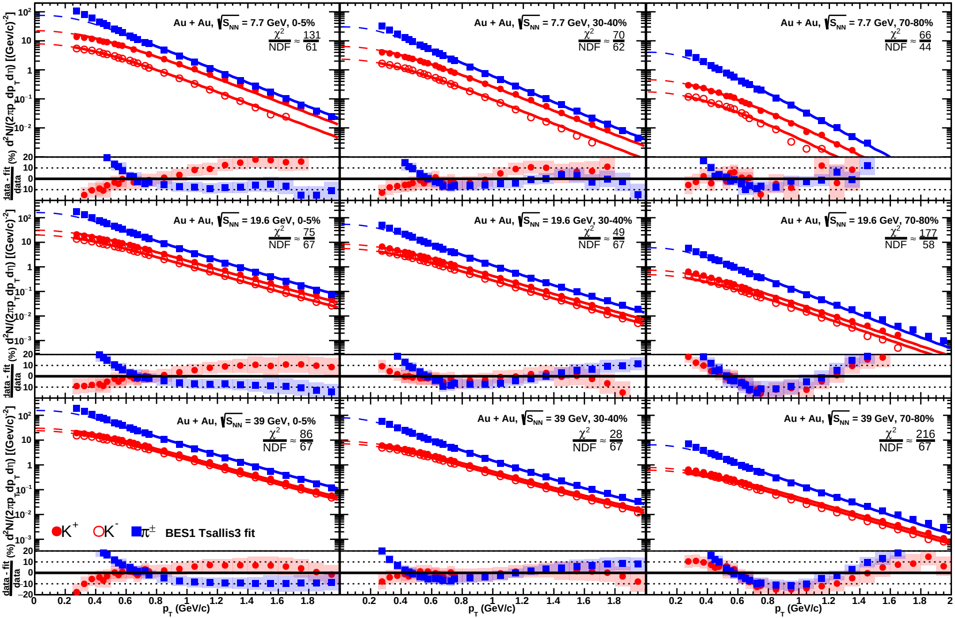
- In this work: $n_0 = 1$, $\mu = 0$.

Tsallis-3 statistics

- For $n_0 = 0$ norm system can be solved analytically: $\Lambda = 0, \ \theta = 1$. Then the expression for distribution takes the form:

$$\left.\frac{\mathrm{d}^2 N}{p_T \, \mathrm{d}p_T \, \mathrm{d}y}\right|_{y_{min}}^{y_{max}} = \frac{gV}{(2\pi)^2} \int\limits_{y_{min}}^{y_{max}} \mathrm{d}y \, m_T \cosh y \left(1 - (1-q)\frac{m_T \cosh y - \mu}{T}\right)^{\frac{q}{1-q}}$$

- This expression reproduces the phenomenological Tsallis distribution.
- Other kinds of distributions are thermodynamically inconsistent, as shown in [Parvan A.S., 10.1088/1751-8121/ac0ebd], because there the mean of the values is incorrectly calculated, which leads to $\langle 1 \rangle \neq 1$. **If we want to obtain physically meaningful thermodynamic parameters, we should use thermodynamically consistent distributions.**



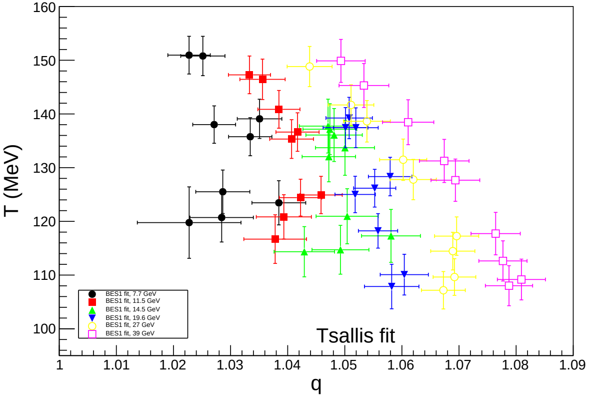
Au + Au, √S_NN = 7.7 GeV, 0-5%
χ²/NDF ≈ 131/61
Au + Au, √S_NN = 7.7 GeV, 30-40%
χ²/NDF ≈ 70/62
Au + Au, √S_NN = 7.7 GeV, 70-80%
χ²/NDF ≈ 66/44
Au + Au, √S_NN = 19.6 GeV, 0-5%
χ²/NDF ≈ 75/67
Au + Au, √S_NN = 19.6 GeV, 30-40%
χ²/NDF ≈ 49/67
Au + Au, √S_NN = 19.6 GeV, 70-80%
χ²/NDF ≈ 177/58
Au + Au, √S_NN = 39 GeV, 0-5%
χ²/NDF ≈ 86/67
Au + Au, √S_NN = 39 GeV, 30-40%
χ²/NDF ≈ 28/67
Au + Au, √S_NN = 39 GeV, 70-80%
χ²/NDF ≈ 216/67
K⁺ K⁻ π± BES1 Tsallis3 fit
d²N/(2πp_T dp_T dη) [(GeV/c)⁻²]
(data - fit)/data (%)
p_T (GeV/c)

Fit results (7.7 GeV)

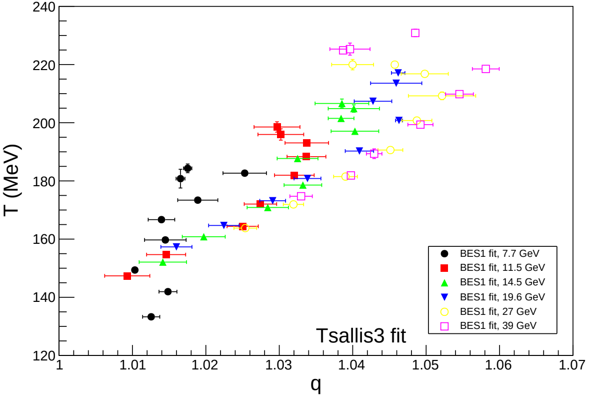
Centrality	T (MeV)	q	χ^2/NDF
0–5%	187.5 ± 1.5	1.0221 ± 0.0022	$131/61$
5–10%	184.9 ± 1.7	1.0227 ± 0.0031	$93/63$
10–20%	186.9 ± 1.1	1.0346 ± 0.0034	$67/63$
20–30%	178.2 ± 1.0	1.0331 ± 0.0029	$73/63$
30–40%	170.5 ± 0.8	1.0277 ± 0.0030	$70/62$
40–50%	161.7 ± 0.9	1.0326 ± 0.0029	$65/57$
50–60%	151.6 ± 0.9	1.0206 ± 0.0031	$81/57$
60–70%	143.8 ± 1.0	1.0157 ± 0.0032	$73/53$
70–80%	136.8 ± 0.9	1.0082 ± 0.0041	$66/44$

Fit results

Zeroth term approximation



Exact Tsallis-3 statistics



General Relations

- Assume that particles are emitted from a fireball expanding longitudinally with rapidity η and transversely with some radial rapidity $\rho = \tanh^{-1}\beta(r)$. Full velocity field takes the form:

$$u^{\nu}(\rho,\ \eta) = (\cosh(\rho)\cosh(\eta),\ \vec{e_r}\sinh(\rho),\ \cosh(\rho)\sinh(\eta)).$$

- The transverse momentum spectrum is given by the Cooper-Frye formula:

$$\frac{\mathrm{d}^3N}{p_T\,\mathrm{d}p_T\,\mathrm{d}y\,\mathrm{d}\varphi} = \frac{1}{(2\pi)^3}\int\limits_{\Sigma}\mathrm{d}\Sigma_{\lambda}\,p^{\lambda}\sum_{\sigma}\langle n_{\vec{p}\sigma}\rangle$$

where Σ - the four-dimensional freeze-out hypersurface.

- Parameterizing the hypersurface $\Sigma(r,\ \phi,\ \zeta)$ in cylindrical coordinates:

$$r \in [0,\ R],\ \phi \in [0,\ 2\pi],\ \zeta \in [-Z, Z].$$

General Relations

- Thus, we obtain:

$$\frac{\mathrm{d}^3 N}{p_T\,\mathrm{d}p_T\,\mathrm{d}\,y\,\mathrm{d}\,\varphi} = \frac{1}{(2\pi)^3}\int\limits_0^R r\,\mathrm{d}r\int\limits_0^{2\pi}\mathrm{d}\phi\int\limits_{-Z}^{Z}\mathrm{d}z\,m_T(\cosh y - \sinh y\tanh\eta)\sum_\sigma\langle n_{\vec{p}\sigma}\rangle$$

- where we include expansion flow in occupation numbers:

$$E_{\vec{p}} = u^\mu p_\mu = m_T\cosh(y-\eta)\cosh\rho - p_T\sinh\rho\cosh(\varphi-\phi)$$

Boltzmann-Gibbs statistics

- Boltzmann-Gibbs statistics:

$$\langle n_{\vec{p}\sigma} \rangle = \frac{1}{\exp\left(\frac{u^{\mu}p_{\mu}-\mu}{T}\right)+\eta},$$

where $\eta = -1$(Bose–Einstein), 0(Maxwell–Boltzmann), 1(Fermi–Dirac).

- For $\eta = 0$(Maxwell–Boltzmann) transverse momentum distribution can be integrated by y and ϕ analytically:

$$\frac{\mathrm{d}N}{p_T\,\mathrm{d}p_T} = \frac{gV}{4\pi^3} m_T e^{\frac{\mu}{T}} \int_0^1 \tilde{r}\,\mathrm{d}\tilde{r}\,\mathrm{K}_1\left(\frac{m_T\cosh\rho}{T}\right)\mathrm{I}_0\left(\frac{p_T\sinh\rho}{T}\right),$$

where $V = \pi R^2 \tau 2\eta_{\mathsf{max}}$ - cylinder volume.

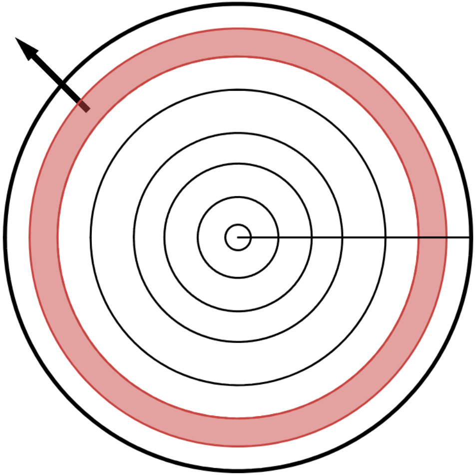
Boltzmann-Gibbs statistics

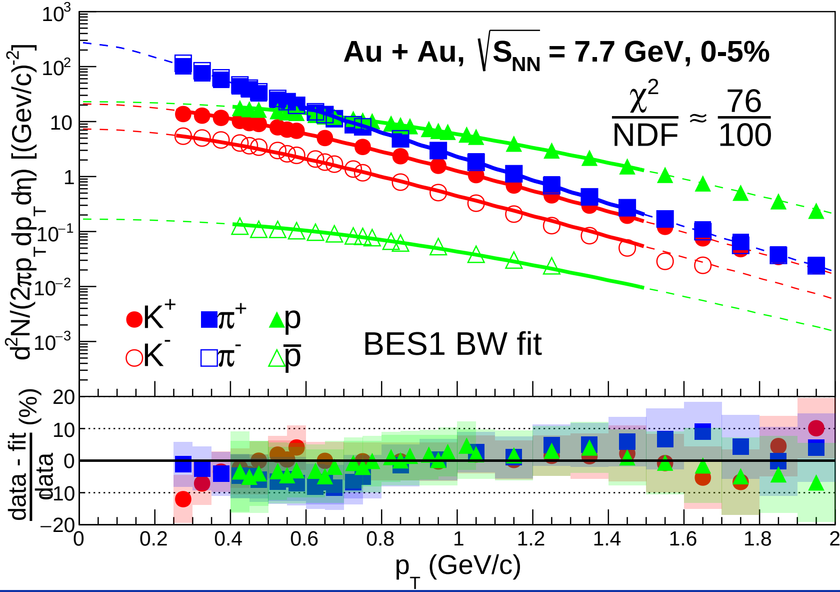
- Expansion rapidity parameterization:

$$\rho(r) = \rho_0 \left(\frac{r}{R}\right)^n,$$

where n defines the transverse flow rapidity profile.

- We use n = 1.





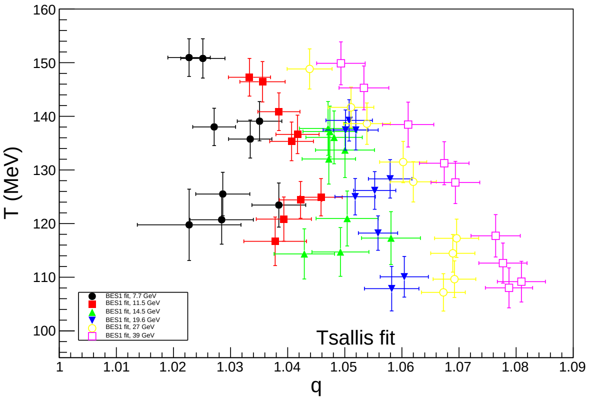
Au + Au, $\sqrt{S_{NN}}$ = 7.7 GeV, 0-5%
$\frac{\chi^2}{NDF} \approx \frac{76}{100}$
BES1 BW fit
K^+
K^-
π^+
π^-
p
$\bar{p}$
$d^2N/(2\pi p_T dp_T dm)$ [(Gev/c)$^{-2}$]
$\frac{data - fit}{data}$ (%)
p_T (GeV/c)

Fit results (7.7 GeV)

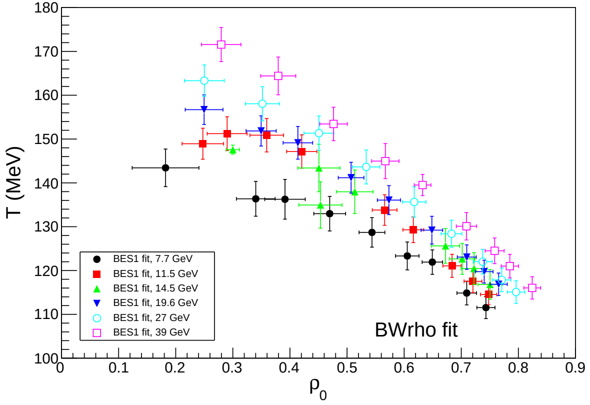
Centrality	ρ_0	T (MeV)	χ^2/NDF
0–5%	0.743 ± 0.016	112 ± 3	$76/100$
5–10%	0.710 ± 0.017	115 ± 3	$64/102$
10–20%	0.649 ± 0.018	122 ± 3	$48/106$
20–30%	0.605 ± 0.021	123 ± 3	$79/104$
30–40%	0.544 ± 0.023	129 ± 3	$81/104$
40–50%	0.470 ± 0.028	133 ± 4	$88/99$
50–60%	0.391 ± 0.035	136 ± 5	$101/97$
60–70%	0.340 ± 0.034	136 ± 4	$80/95$
70–80%	0.182 ± 0.058	143 ± 4	$68/81$

Fit results

Tsallis-3 zeroth term approximation



Blast-wave



Tsallis-3 statistics

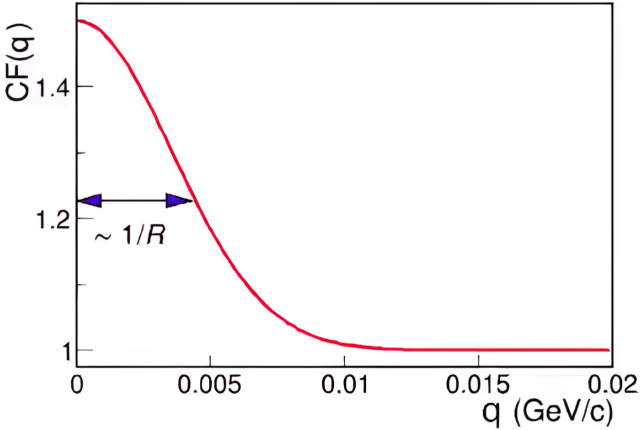
Work in progress...

Introduction to femtoscopy

- Correlation function:

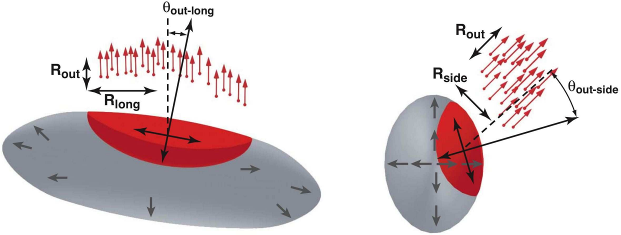
$$C(\vec{q}) = \frac{N(\vec{q})}{D(\vec{q})},$$

  - $N(\vec{q})$ - uses pairs from the same event, contains quantum statistic correlations and interactions,
  - $D(\vec{q})$ - uses pairs from different events, contains only background.



Introduction to femtoscopy

- Bertsch-Pratt out-side-long coordinate system:
  - q_{long} - along the beam direction
  - q_{out} - along the transverse momentum of the pair
  - q_{side} - perpendicular to longitudinal and outward directions



- Gaussian parameterization (Bowler-Sinyukov):

$$C(\vec{q}) = (1-\lambda) + K_{\text{Coul}}(q_{\text{inv}})\lambda \exp\left(-q_{\text{o}}^2 R_{\text{o}}^2 - q_{\text{s}}^2 R_{\text{s}}^2 - q_{\text{l}}^2 R_{\text{l}}^2 - 2q_{\text{o}}q_{\text{s}}R_{\text{os}}^2 - 2q_{\text{o}}q_{\text{l}}R_{\text{ol}}^2\right),$$

  - λ - correlation strength
  - $K_{\text{Coul}}(q_{\text{inv}})$ - Coulomb correction factor
  - $R_{\text{long}} \sim$ source lifetime
  - $R_{\text{out}} \sim$ - geometric size and emission duration
  - $R_{\text{side}} \sim$ - geometric size
  - $R_{\text{os,ol}}$ - crossterms

Volume calculation using correlation radii

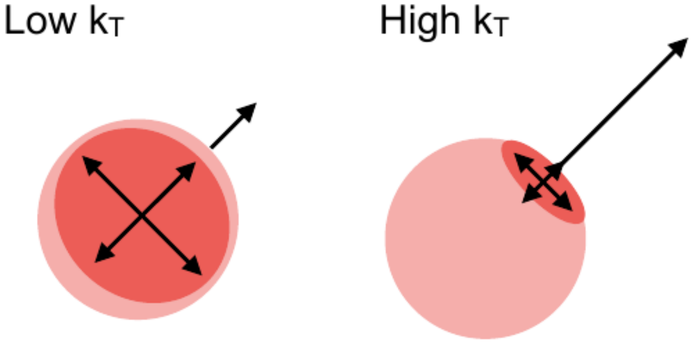
Parameterizations:

- m_T scaling: $R_i \propto \frac{1}{\sqrt{m_T}}$
- Power law: $R_i \propto m_T^{-\alpha}$
- Blast-Wave:

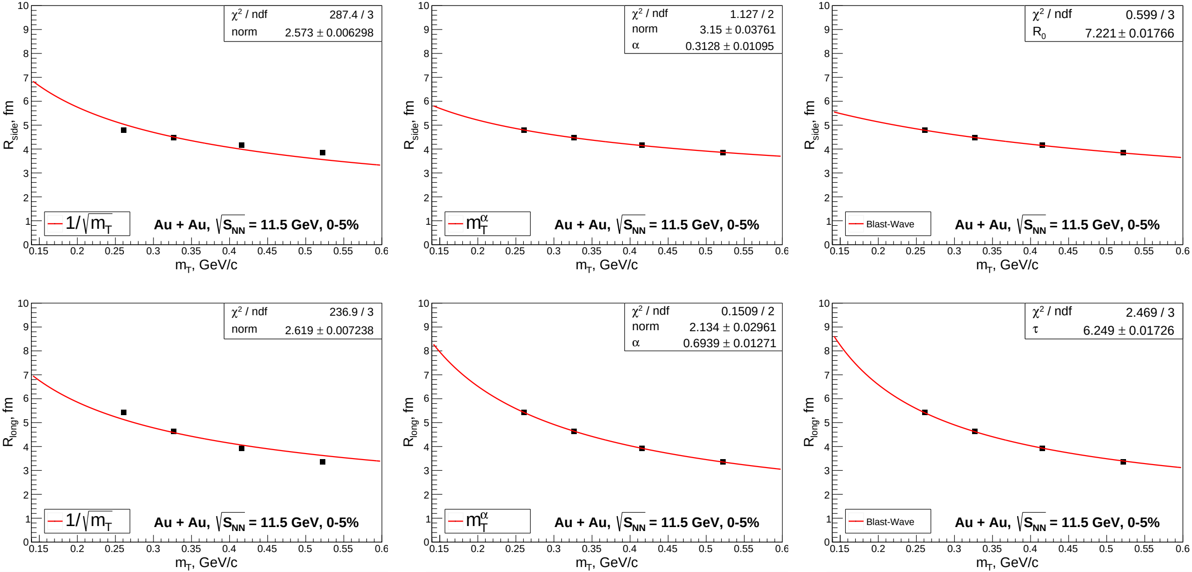
$$R_{\text{side}} = \frac{R_0}{\sqrt{1+\rho_0^2(m_T/T)}},$$

$$R_{\text{long}} = \tau\sqrt{\frac{T}{m_t}\frac{\mathrm{K}_2(m_T/T)}{\mathrm{K}_2(m_T/T)}}$$

(Phys.Rev.C51:328-338 1995)



Volume calculation using correlation radii



Volume calculation using correlation radii

- Recalculate gaussian radii to hard ellipsoid distribution:

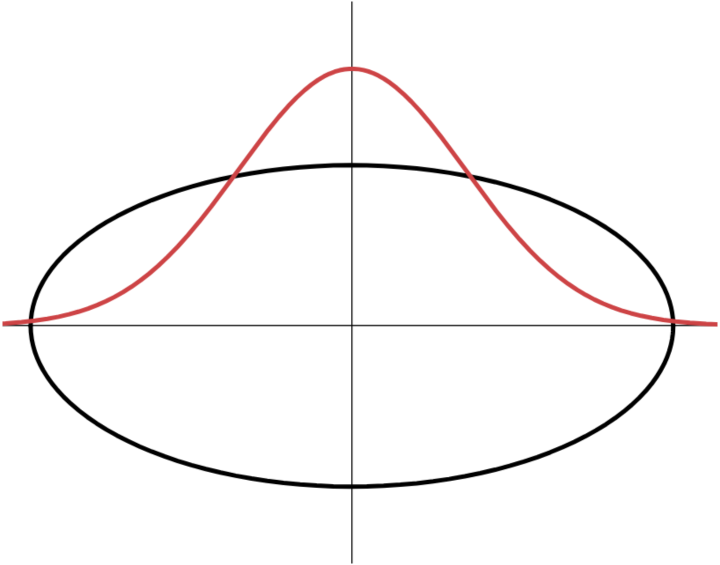
$$\sigma_{\text{Gauss}} = \sigma_{\text{hard}}$$

$$R_{\text{hard}} = \sqrt{5}R_{\text{Gauss}}.$$

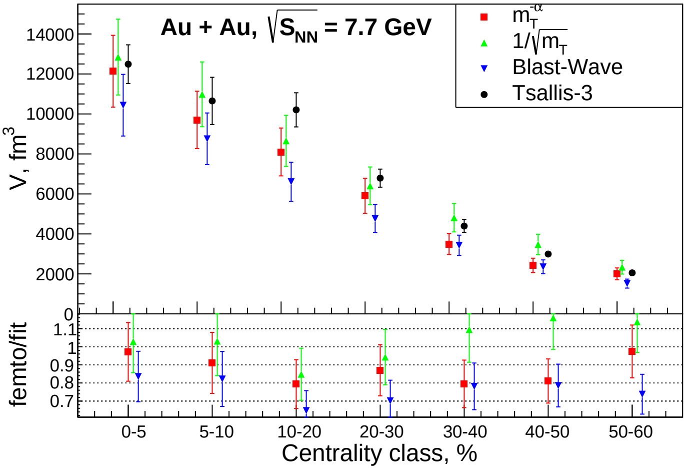
- Calculate the volume:

$$V = \frac{4}{3}\pi \, (5)^{\frac{3}{2}} R_{\text{side}}^2 R_{\text{long}}$$

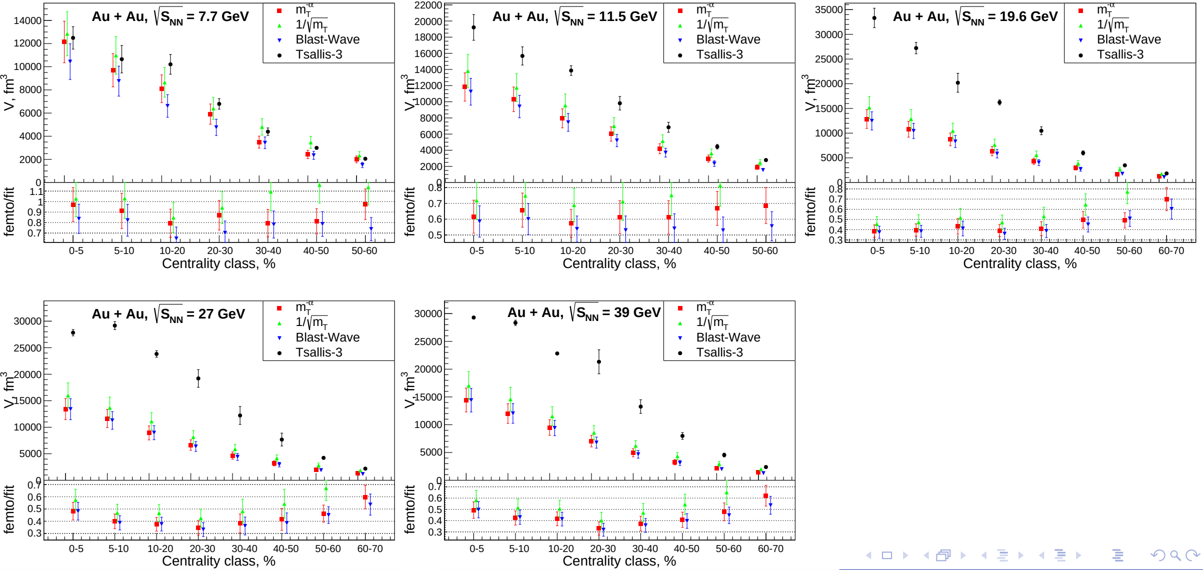
(assumption: $R_{\text{out}} \approx R_{\text{side}}$)



Femtoscopic approach



Femtoscopic approach



Freeze-out cylinder volume calculation

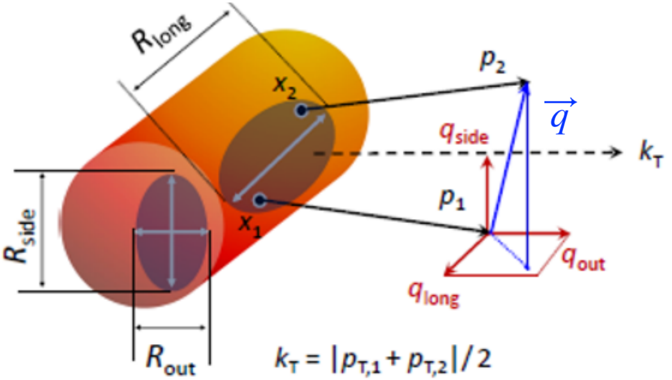
- We can obtain R_{geom} and τ from HBT radii fitting:

$$R_{\text{side}} = \frac{R_{geom}}{\sqrt{1+\rho_0^2(m_T/T)}},$$

$$R_{\text{long}} = \tau\sqrt{\frac{T}{m_T}\frac{\mathrm{K}_2(m_T/T)}{\mathrm{K}_2(m_T/T)}}$$

- Volume of the cylinder:

$$V = \pi R_{geom}^2 \tau 2\eta_{\text{max}}$$



Freeze-out cylinder volume calculation

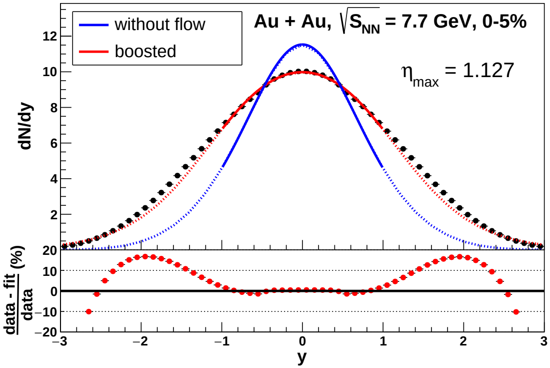
- Longitudinal rapidity η_{max} can be obtained from $\mathrm{d}N/dy$ distributions:

$$\frac{\mathrm{d}N}{\mathrm{d}y\, m_T\, \mathrm{d}m_T\, \mathrm{d}\phi} = \frac{gV}{(2\pi)^3} E e^{-(E-\mu)/T}$$

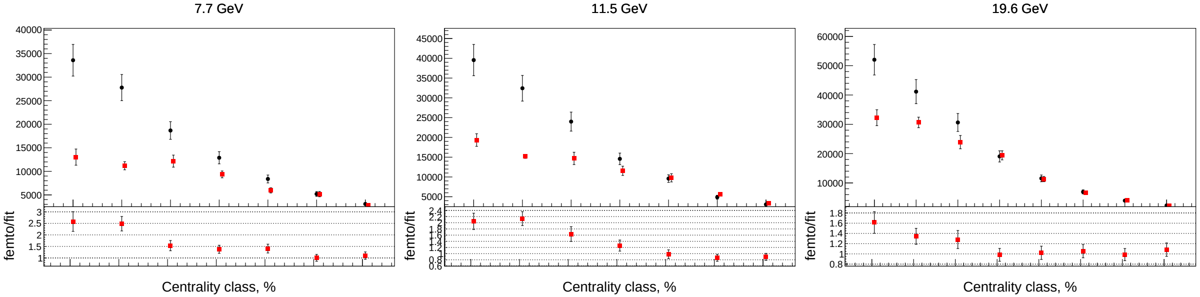
$$\frac{\mathrm{d}N_{th}}{\mathrm{d}y} = \frac{V}{(2\pi)^2} T^3 \left(\frac{m^2}{T^2} + \frac{m}{T}\frac{2}{\cosh y} + \frac{2}{\cosh^2 y} \right) \times$$

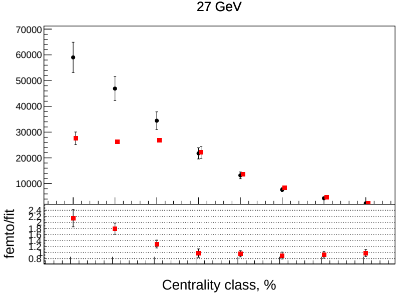
$$\times \exp\left(-\frac{m}{T}\cosh y\right)$$

$$\frac{\mathrm{d}N}{\mathrm{d}y}(y) = \int\limits_{-\eta_{max}}^{\eta_{max}} \mathrm{d}\,\eta \frac{\mathrm{d}N_{th}}{\mathrm{d}y}(y-\eta), \;\; \beta_L = \tanh \eta_{max}$$



Femtoscopic approach





Discussions

- To measure the thermodynamic parameters of the medium at the kinetic freeze-out stage, ideal gas model can be used.
- The Boltzmann-Gibbs statistics do not provide a satisfactory description of the hard part of transverse momentum distributions.
- Tsallis statistics can be introduced from the first principles of thermodynamics.
- The interpretation of the parameter q requires further development.
- The chemical potential must be correctly taken into account.
- The Blast-Wave framework allows one to account for both longitudinal and transverse expansion of the system.
- The volume obtained from spectrum fitting can be compared with the volume obtained from femtoscopy.
- For a more accurate comparison, it is necessary to obtain the Tsallis Blast-Wave and calculate the HBT radii within this formalism.

Thank you for your attention!